Electrical Stimulation of the Heart: The Bidomain Model

A Dozen Examples

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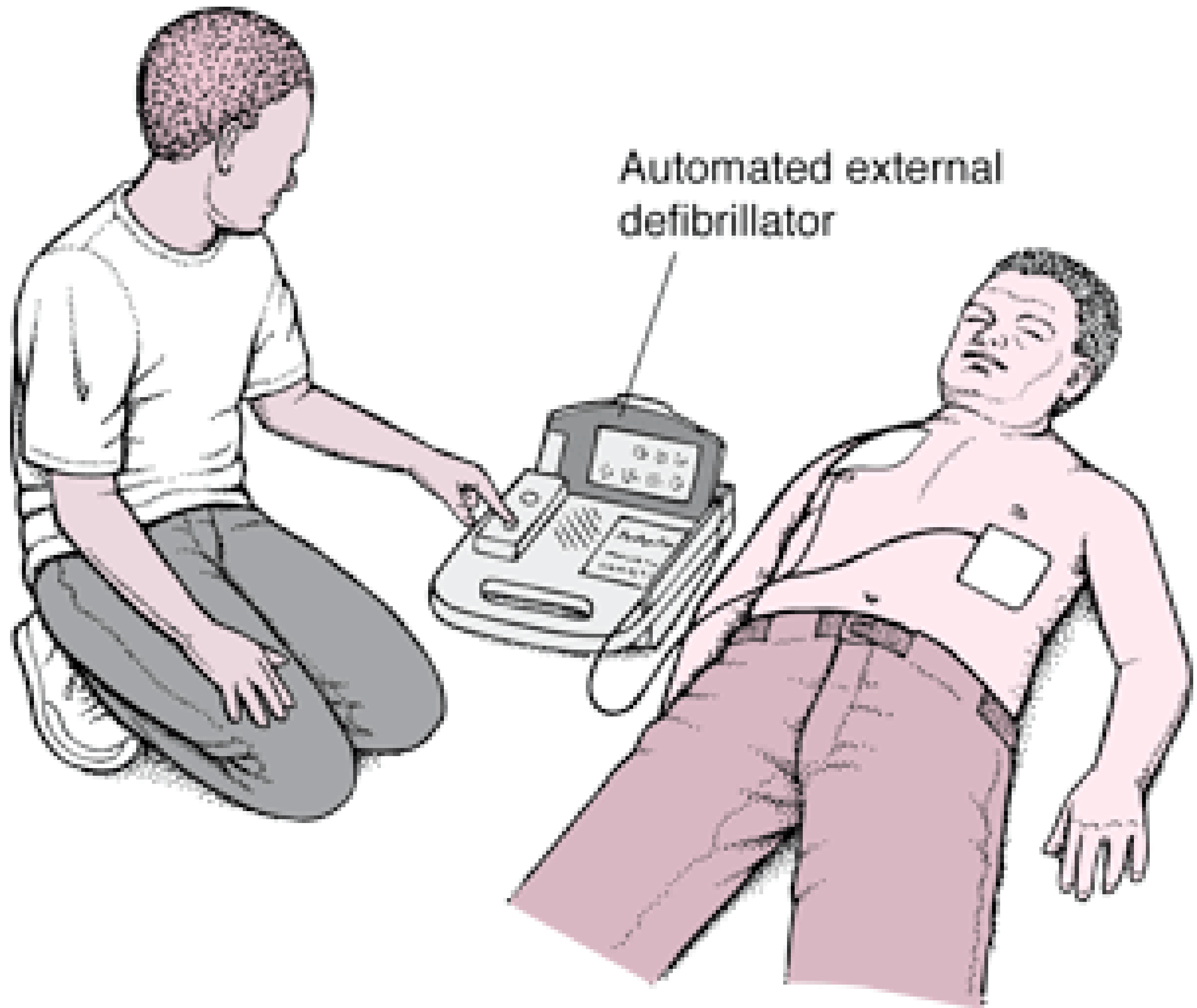


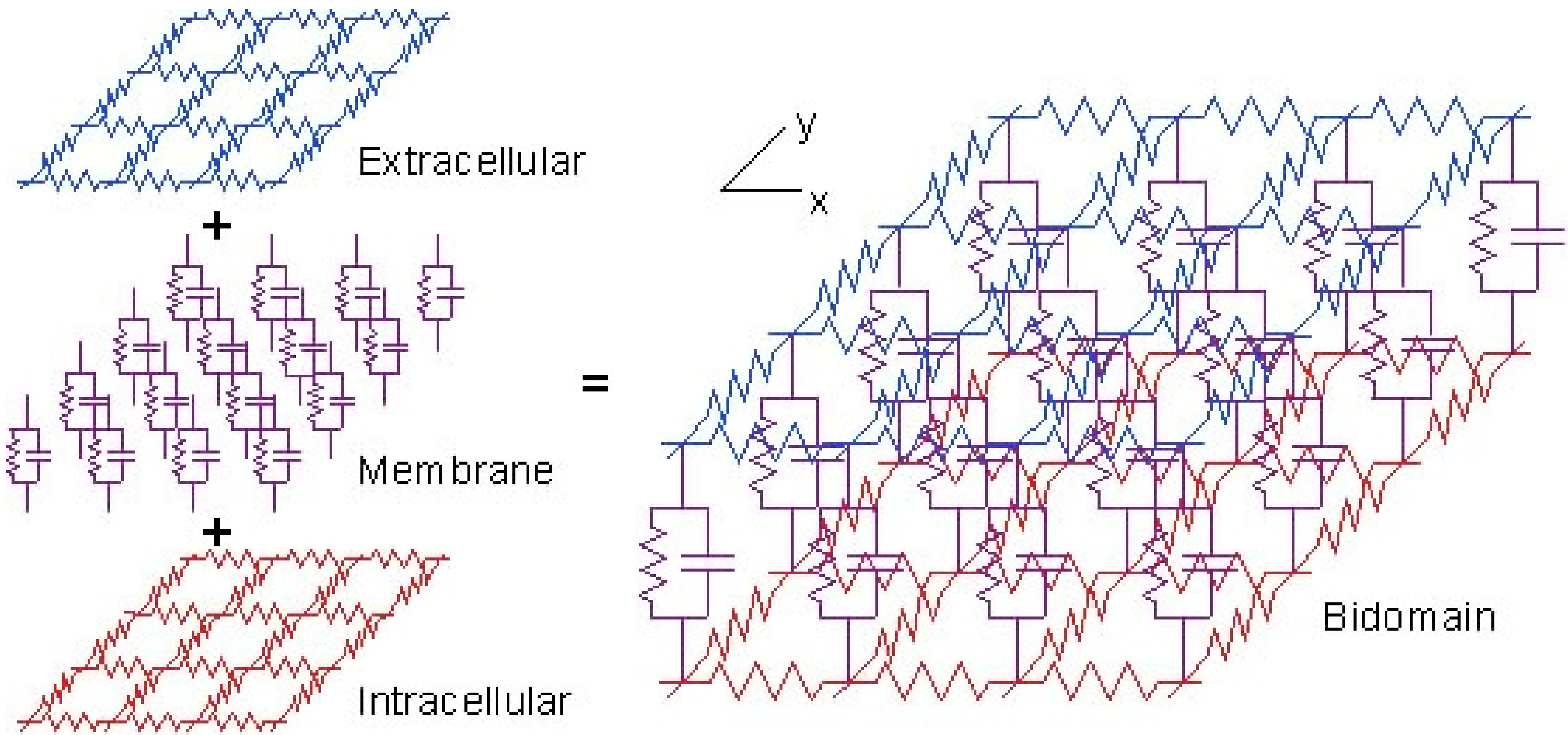
How does an electric field interact with the heart to cause defibrillation?

The Bidomain Model

The **bidomain model** is a set of mathematical equations that govern the electrical properties of cardiac tissue. It was developed in the late 1970s, and is now used extensively in numerical simulations of the electrical behavior of the heart.

http://www.scholarpedia.org/article/The_bidomain_model

The bidomain model is a two- or three-dimensional cable model. It is a continuum model, in the sense that it predicts the electrical behavior averaged over many cells. The model accounts for the different electrical conductivities of the intracellular and extracellular spaces



Both of these spaces are anisotropic: they have a different electrical conductivity in the direction parallel to the myocardial fibers than in the direction perpendicular to them

Ohm's law,   V=IR

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Current Density   Conductivity   Electric Field

If you align the x axis with the fiber direction, the off-diagonal terms are zero

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Current Density　　　　Conductivity　　　　Electric Field

Or in other words….

$$J_x = \sigma_x E_x$$

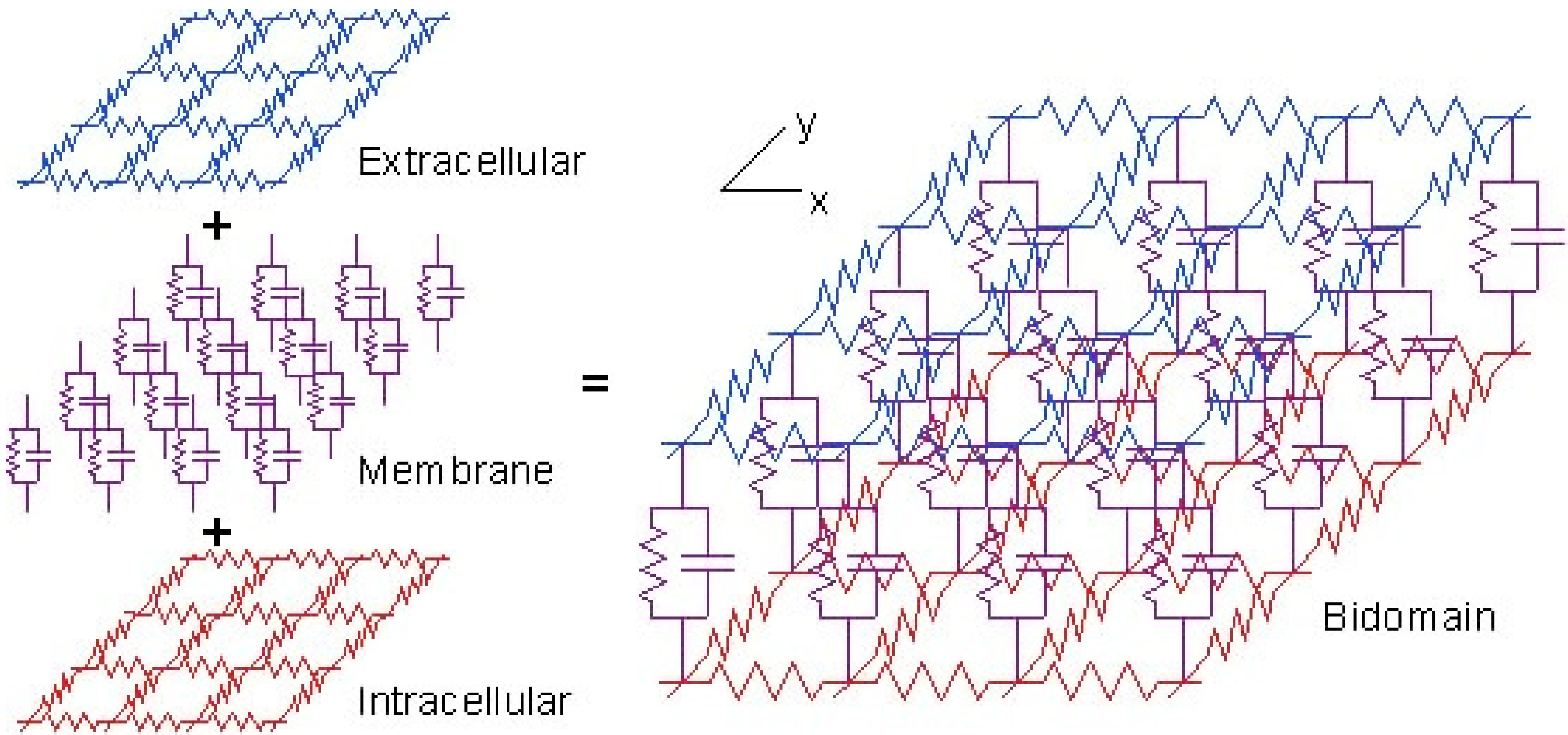
$$J_y = \sigma_y E_y$$

There are two spaces—intracellular and extracellular—so there are four conductivity parameters

σ_{ix}	intracellular conductivity parallel to the fibers	0.20 S/m
σ_{iy}	intracellular conductivity perpendicular to the fibers	0.02 S/m
σ_{ex}	extracellular conductivity parallel to the fibers	0.20 S/m
σ_{ey}	extracellular conductivity perpendicular to the fibers	0.08 S/m

Roth, “Electrical conductivity values used with the bidomain model of cardiac tissue” IEEE Trans. Biomed. Eng., 44:326-328, 1997.

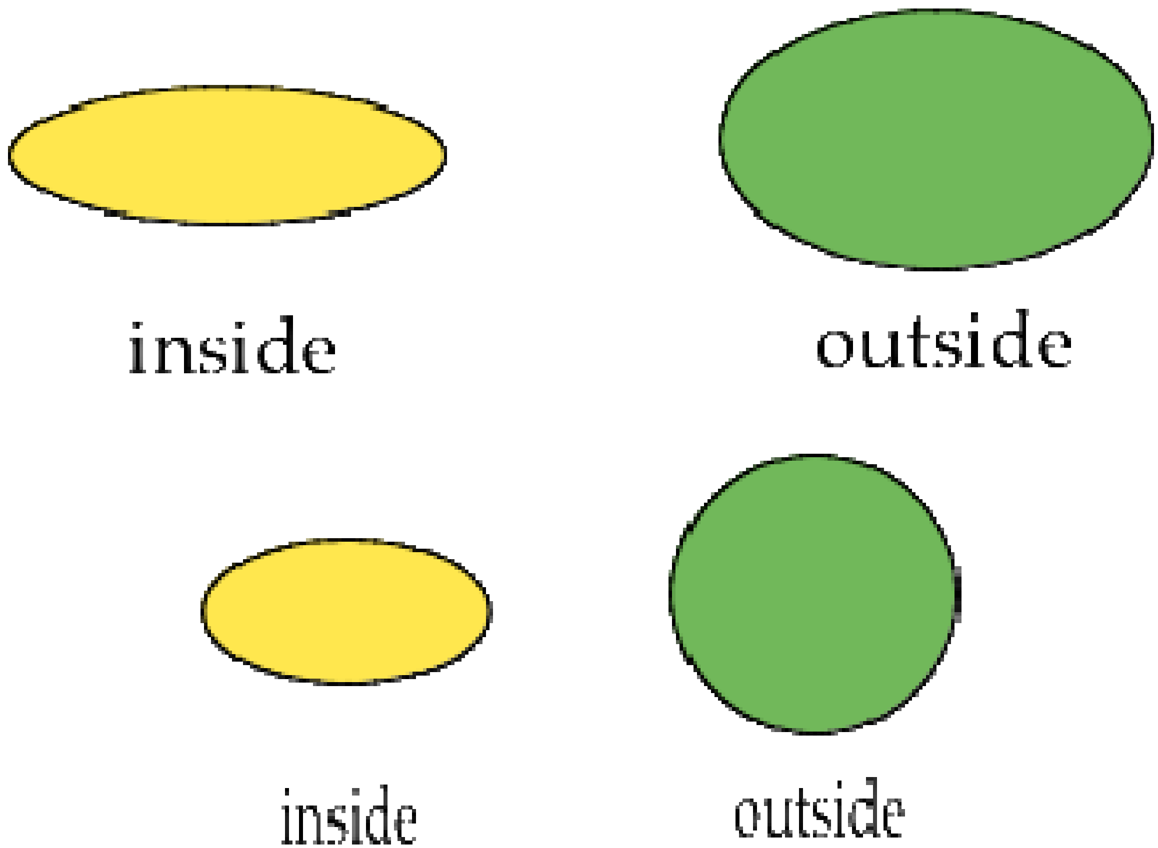
Resisters in the x direction are smaller than resisters in the y direction



Unequal anisotropy ratios

$$\frac{\left(\dfrac{\sigma_{ix}}{\sigma_{iy}}\right)}{\left(\dfrac{\sigma_{ex}}{\sigma_{ey}}\right)} \neq 1$$

Simultaneous Diagonalization of Two Quadratic Forms

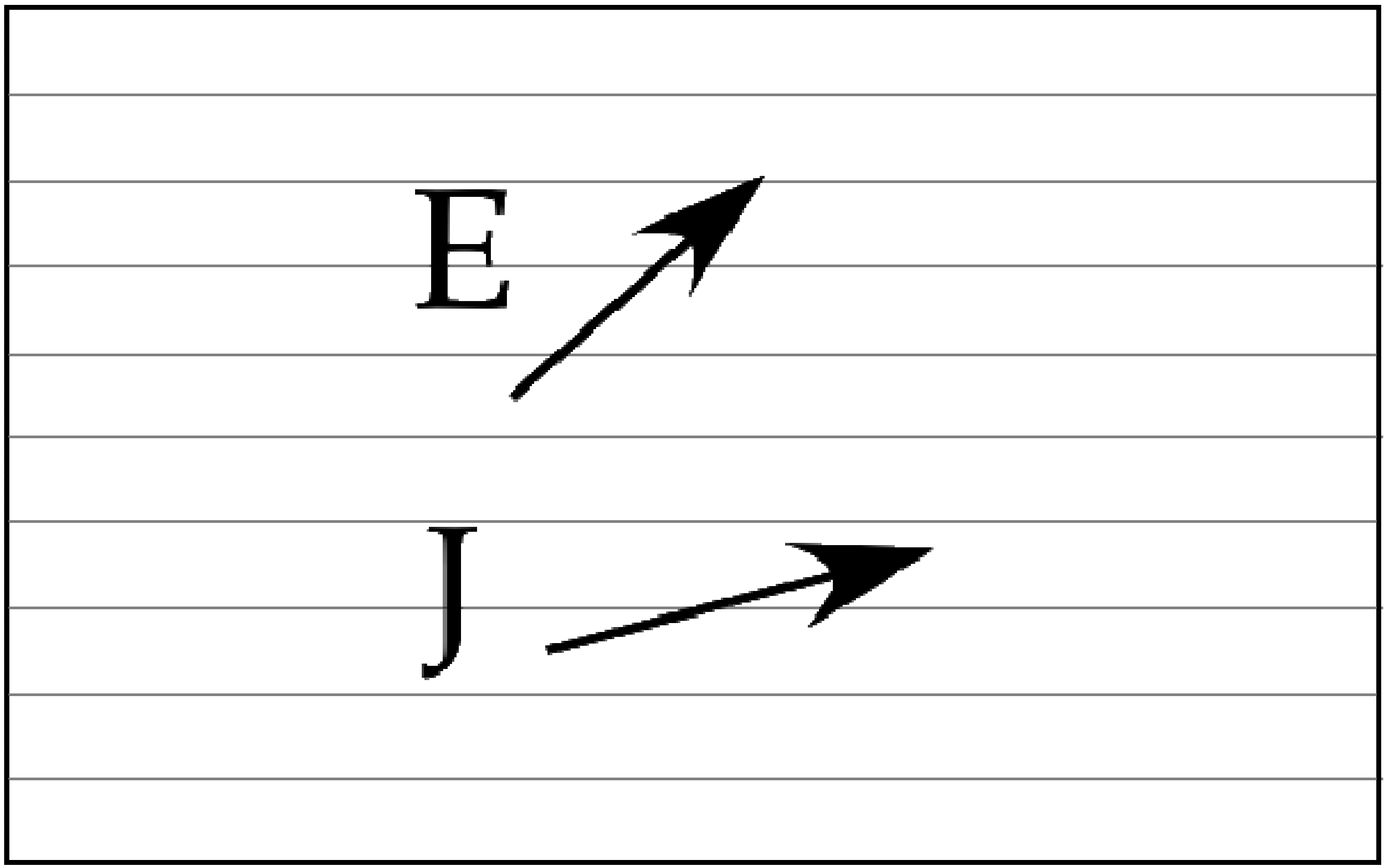


Two Mechanisms

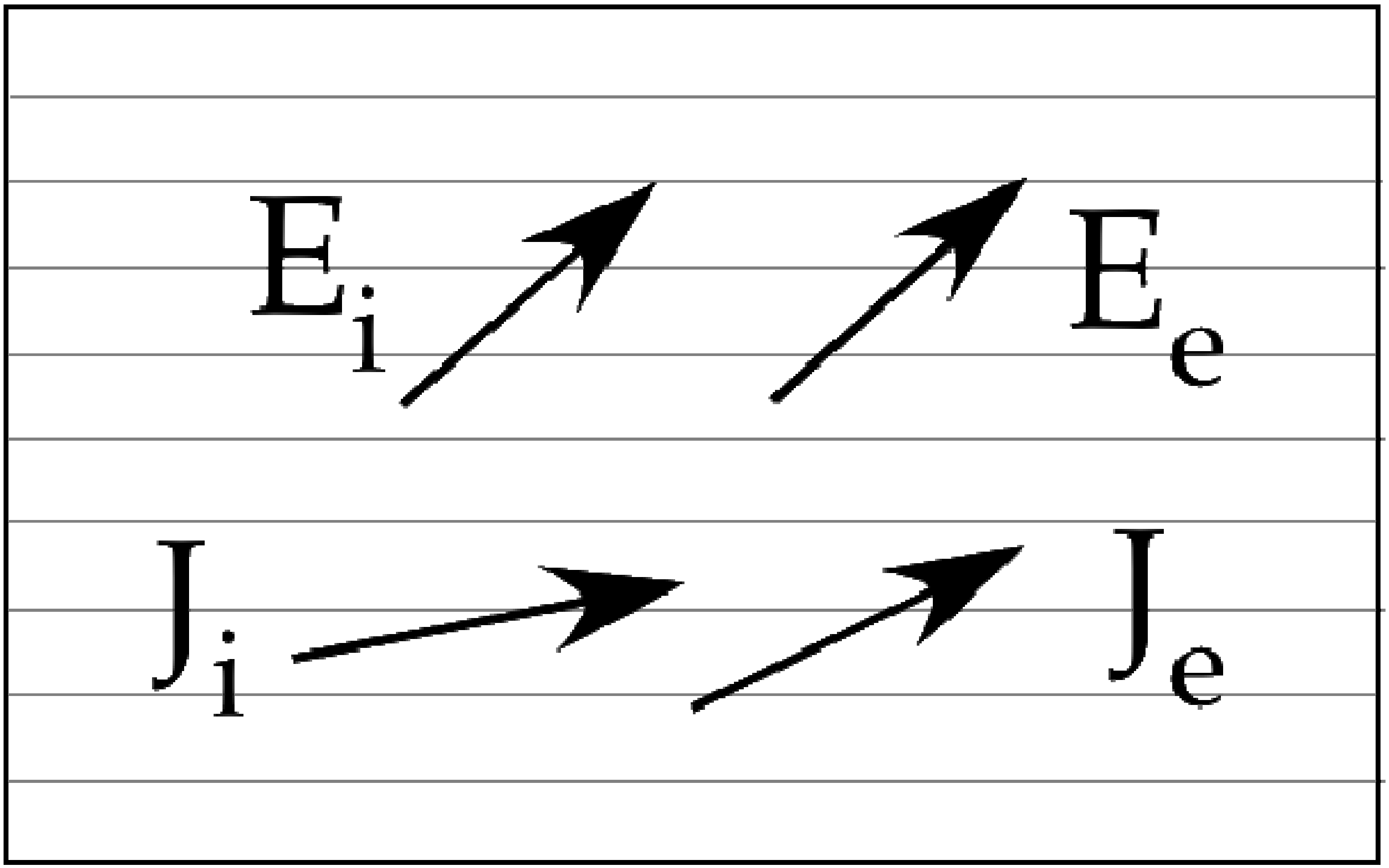
1) Rotation

2) Distribution

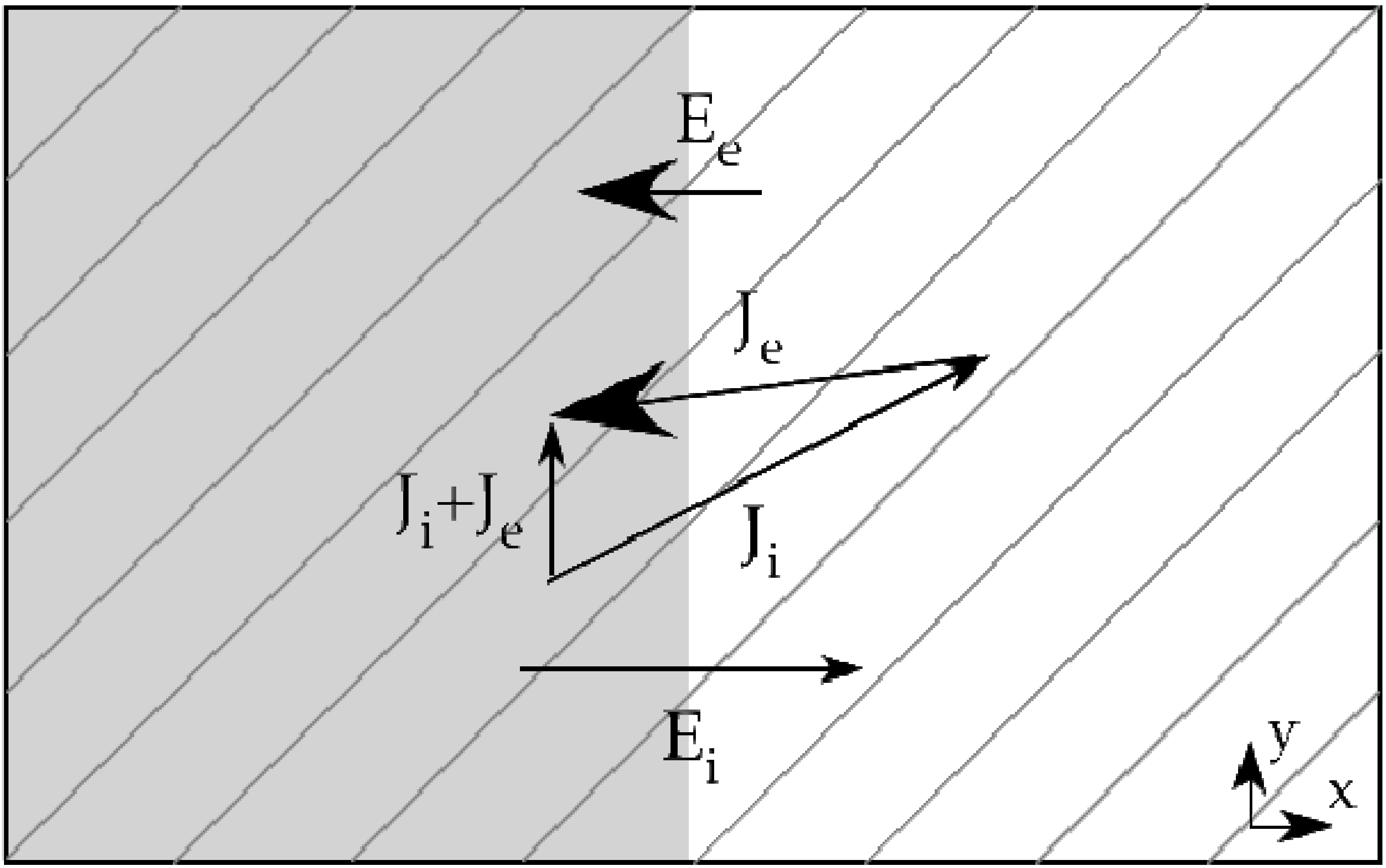
Rotation



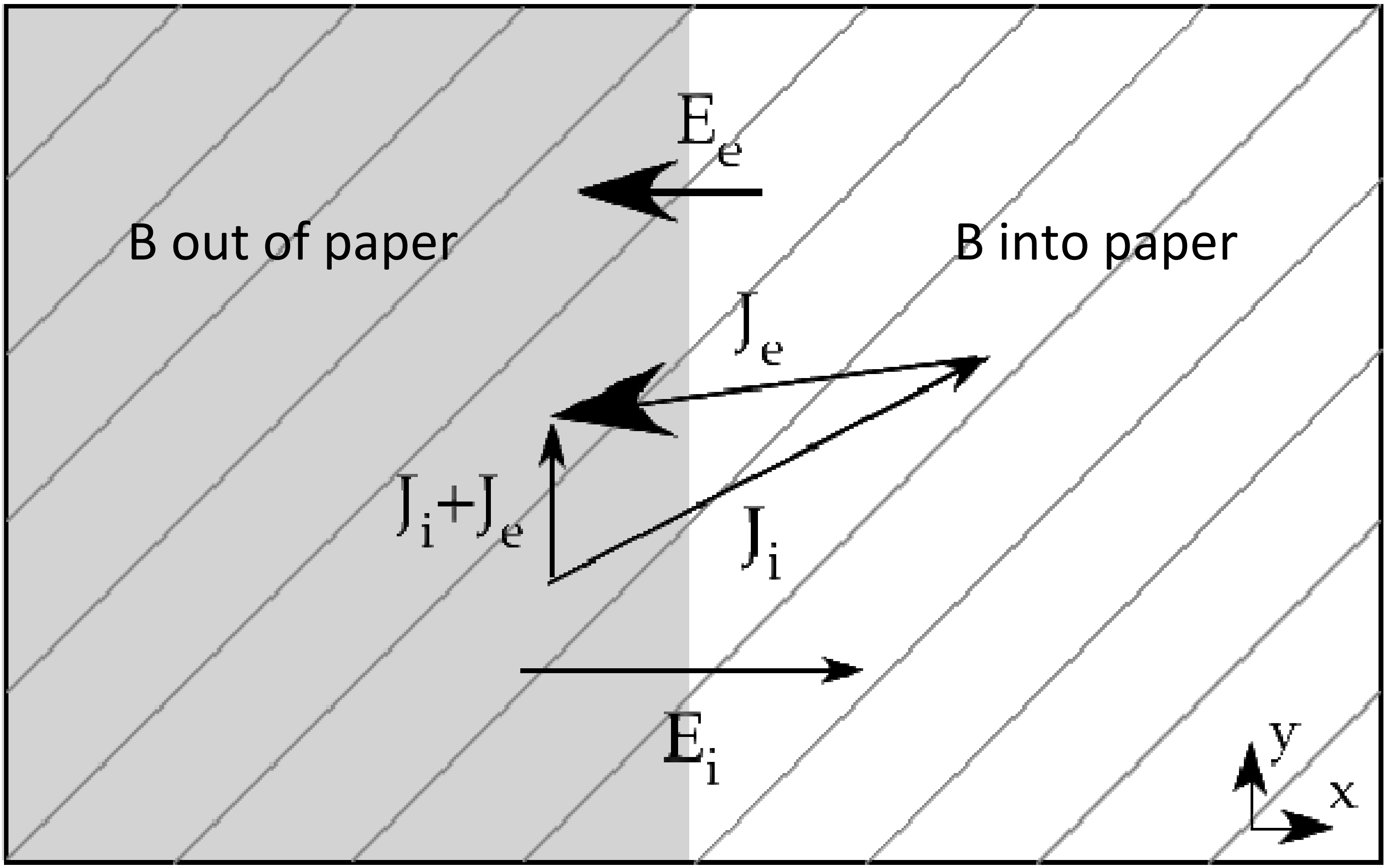
Unequal Anisotropy Ratios



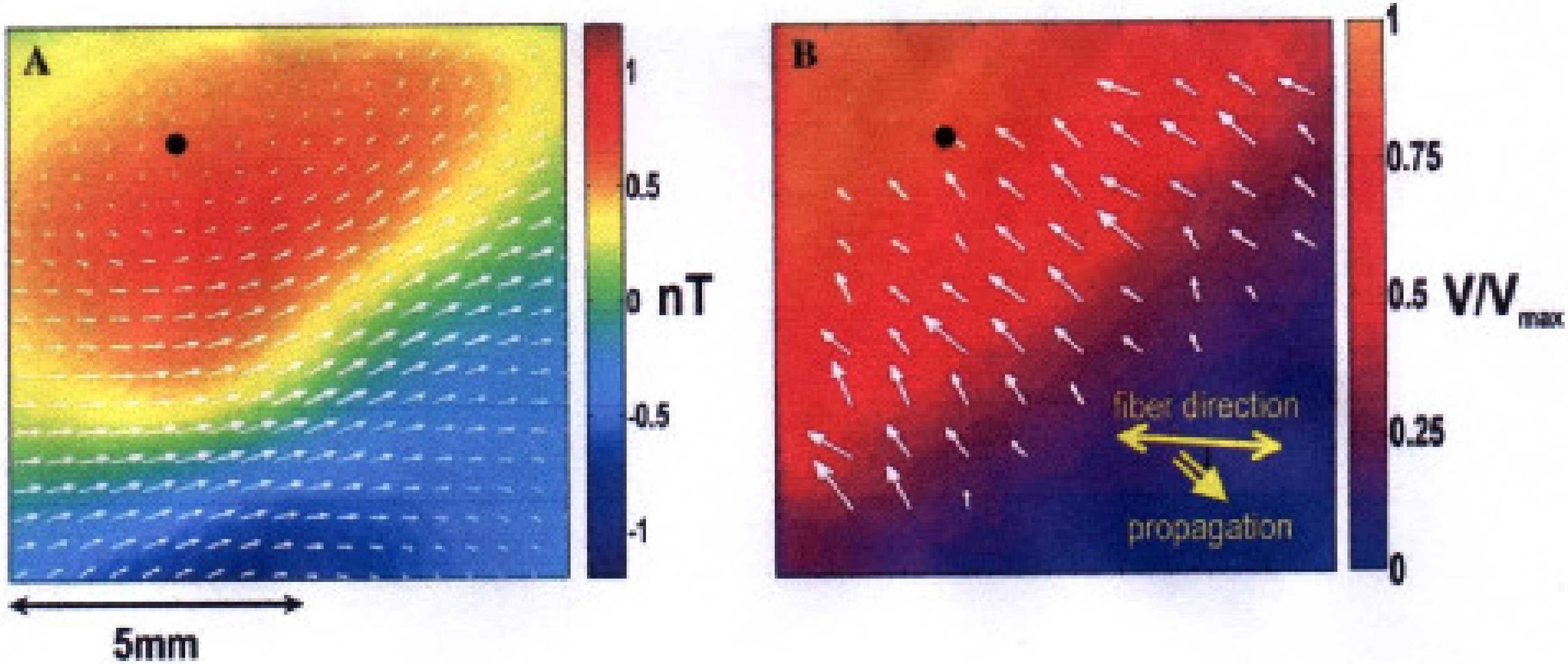
Example 1: The magnetic field of an action potential wave front


E_e
J_e
J_i+J_e
J_i
E_i
y
x

Currents produce a magnetic field

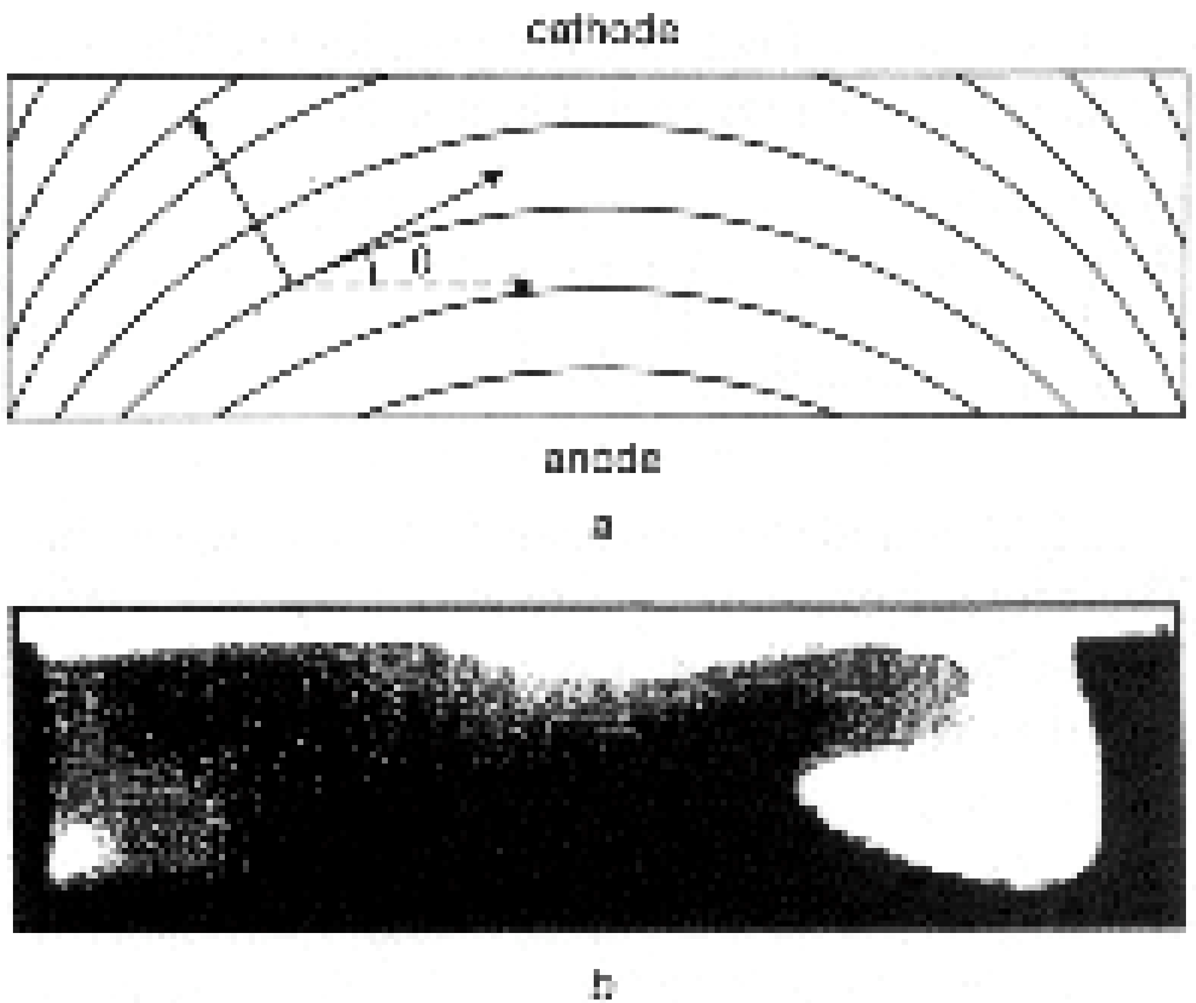


Roth and Woods, 1999, “The magnetic field associated with a plane wave front propagating through cardiac tissue” IEEE Trans. Biomed. Eng., 46:1288-1292, 1999.

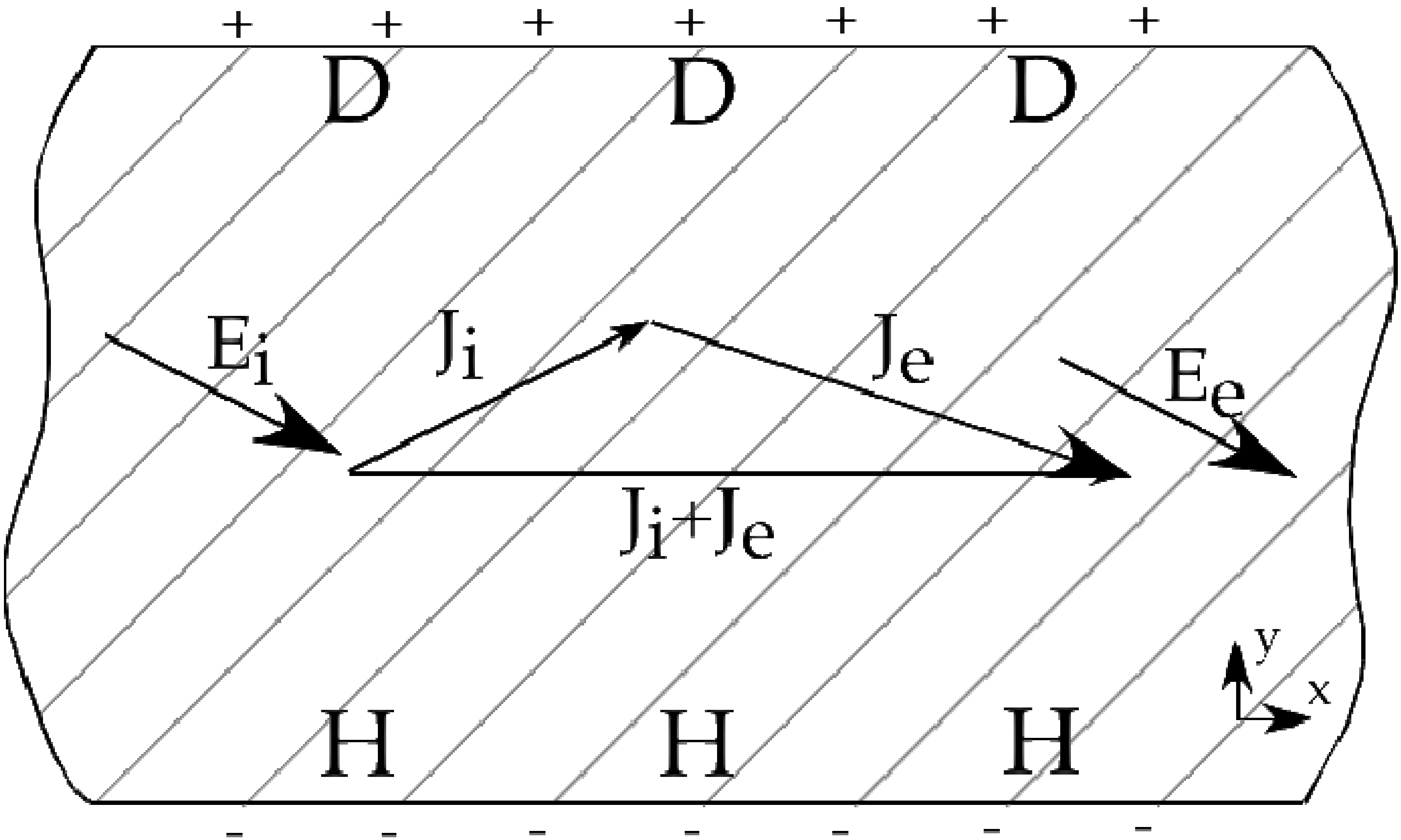


Holzer et al., “High Resolution Magnetic Images of Planar Wave Fronts Reveal Bidomain Properties of Cardiac Tissue” Biophys. J., 87:4326–4332, 2004

Example 2: The transmembrane induced near an insulating boundary

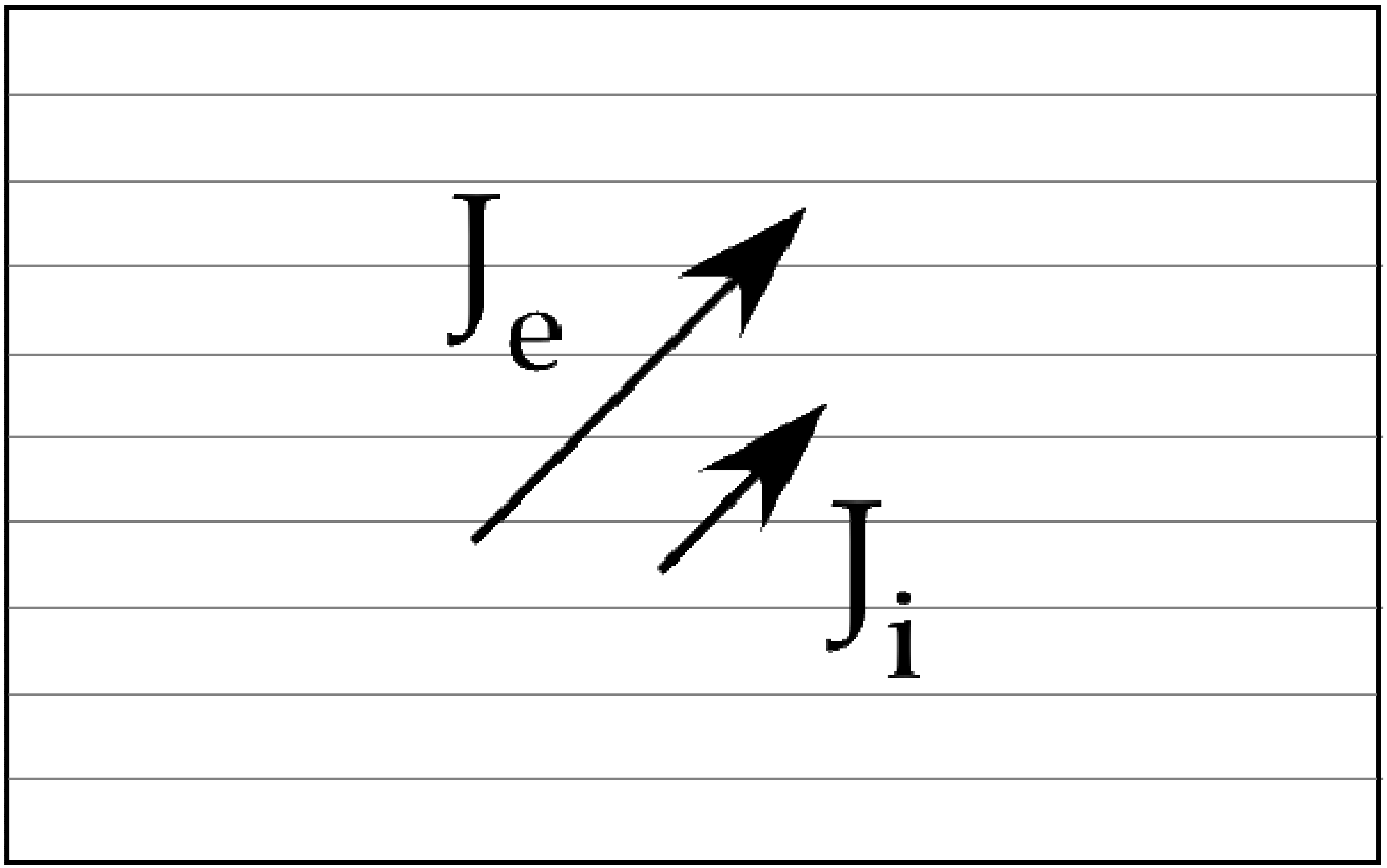


Skouibine, Trayanova and Moore, “Success and failure of the defibrillation shock: Does it depend on the fiber field?” 20th Annual International Conference of the IEEE EMBS, October 29-November 1, Hong Kong, 1998.

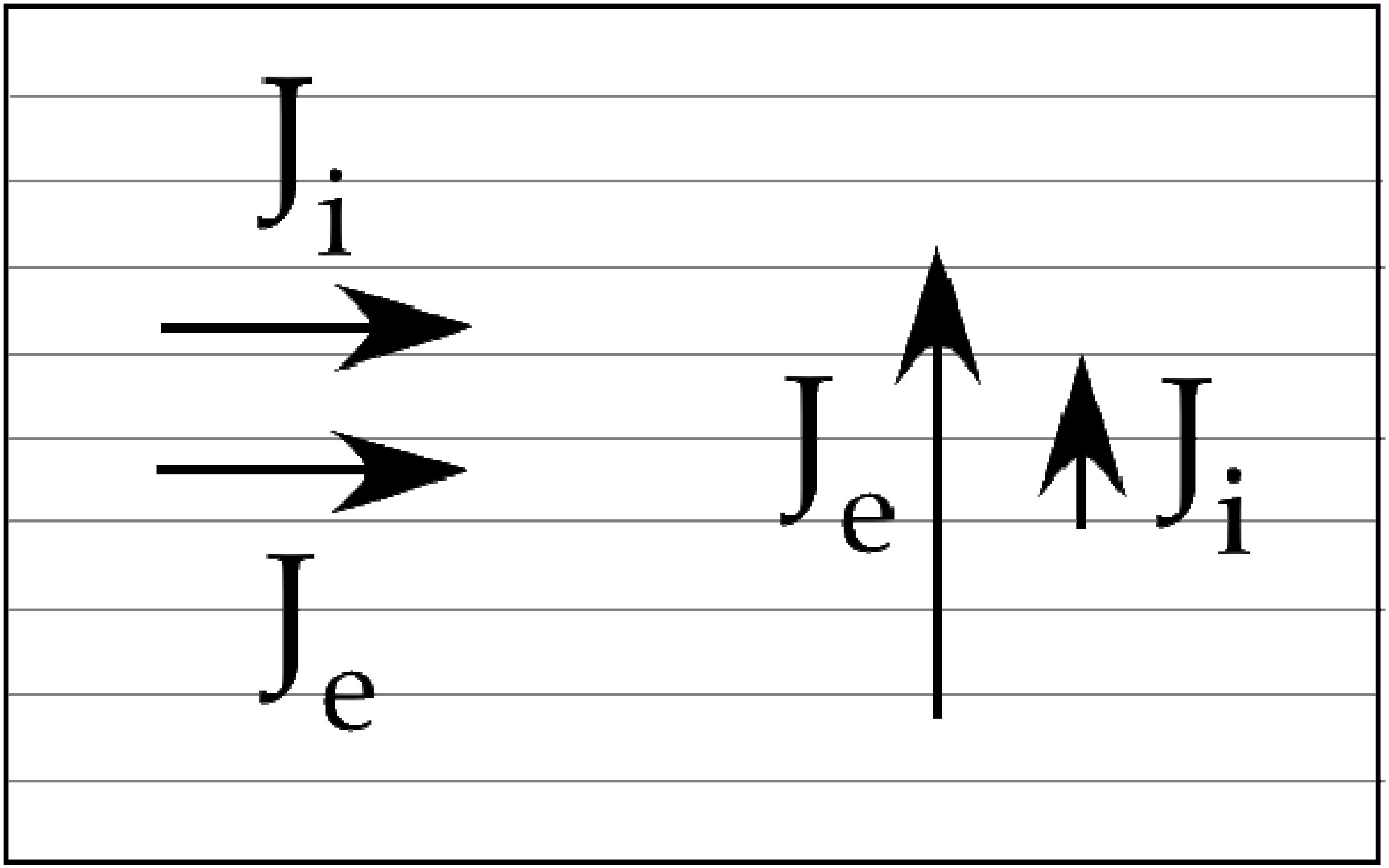


Roth, “Mechanism for polarization of cardiac tissue near a sealed boundary” Med. & Biol. Eng. & Comput., 37:523–525, 1999.

Distribution

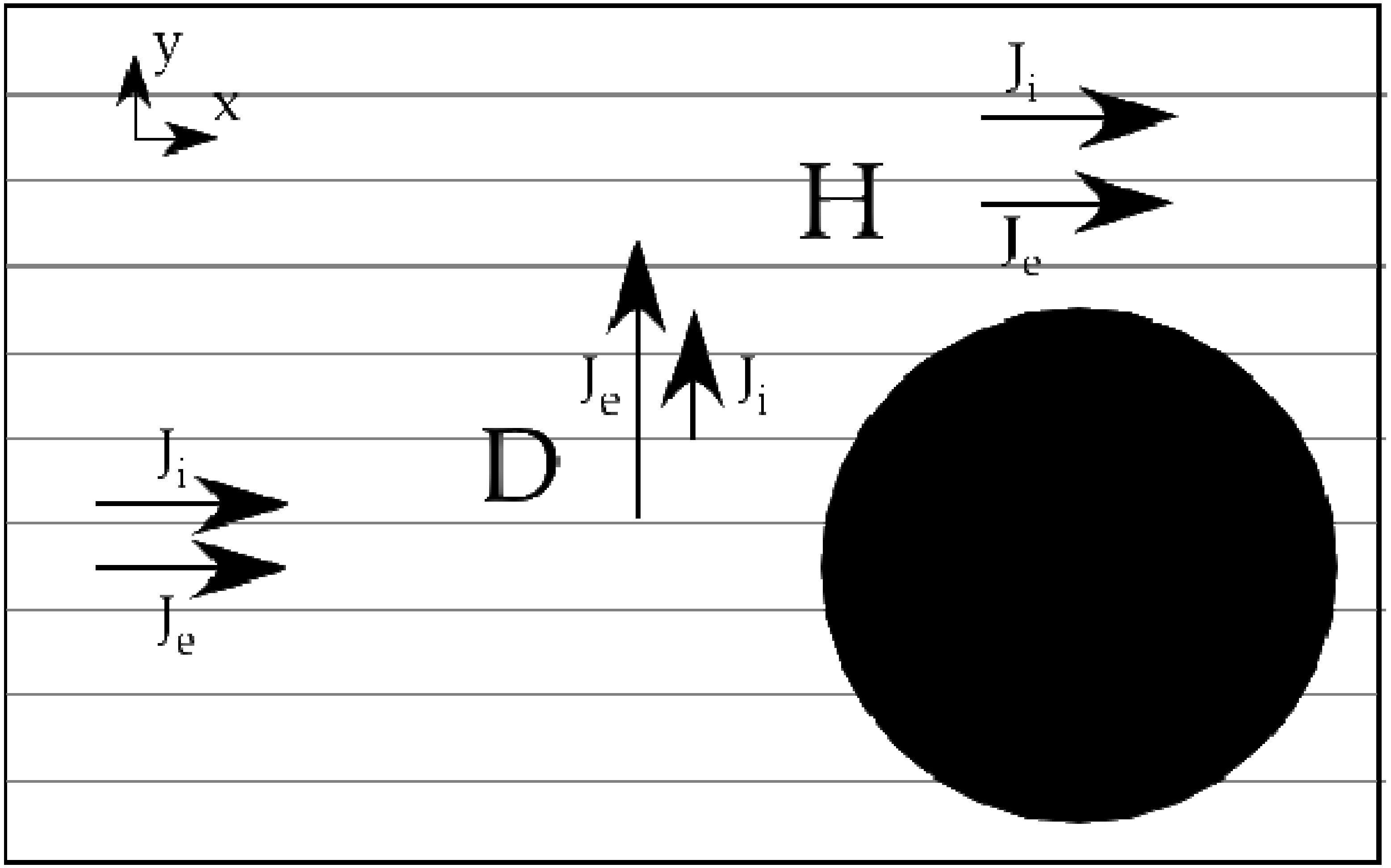


Unequal Anisotropy Ratios

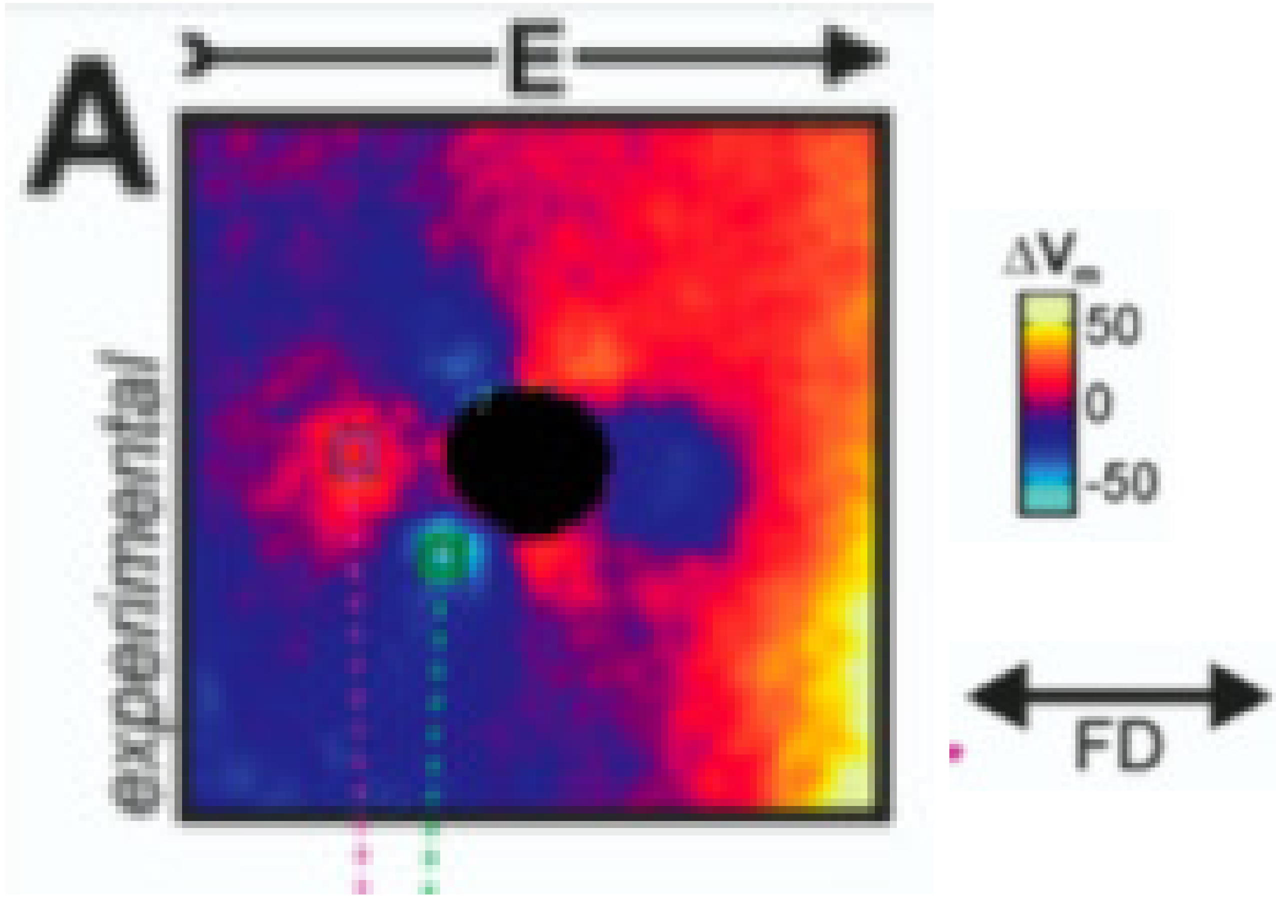


Example 3: The transmembrane potential induced around a circular insulator

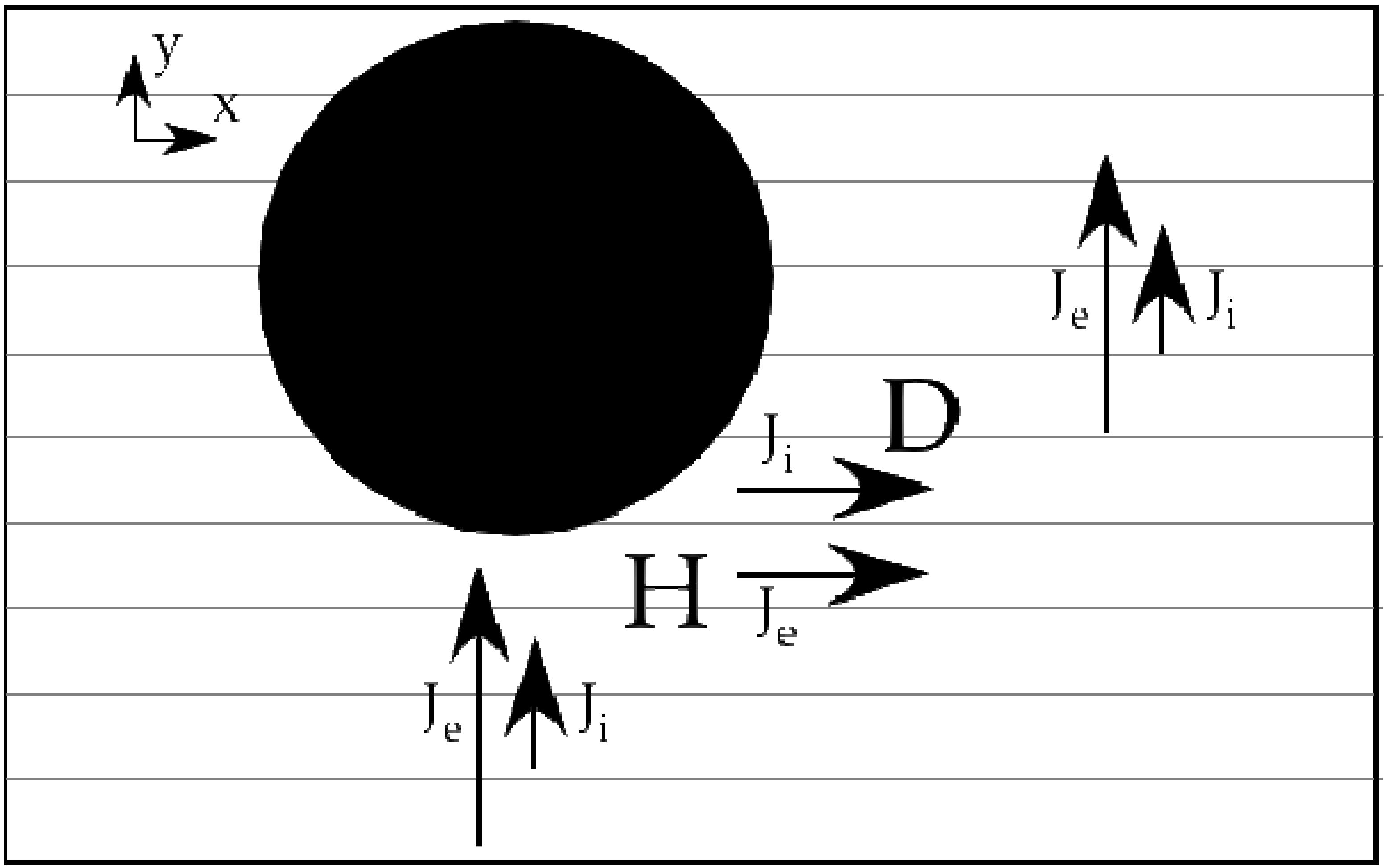
Transmembrane potential around an insulator



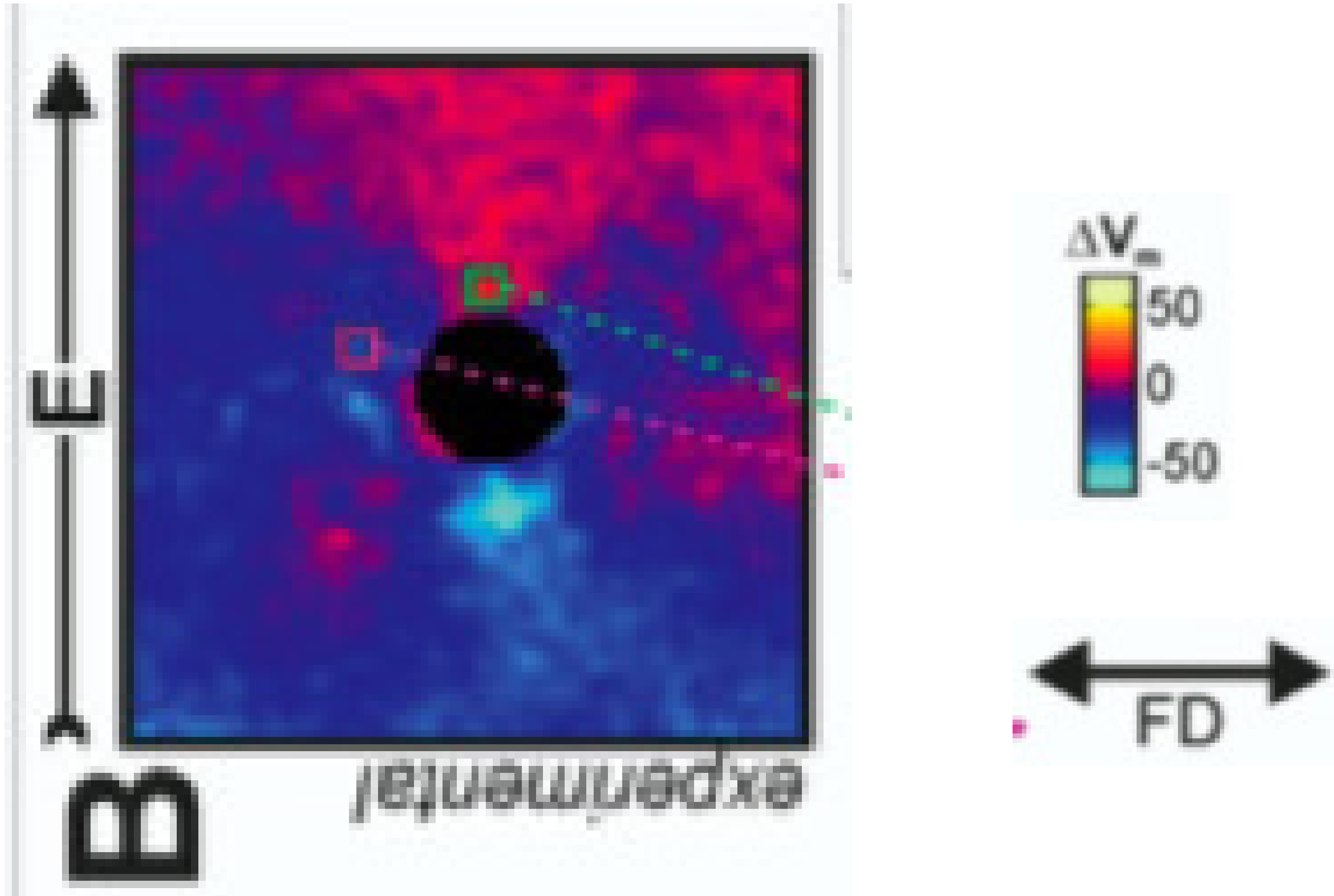
Langrill and Roth, “The effect of plunge electrodes during electrical stimulation of cardiac tissue” IEEE Trans. Biomed. Eng., 48:1207-1211, 2001



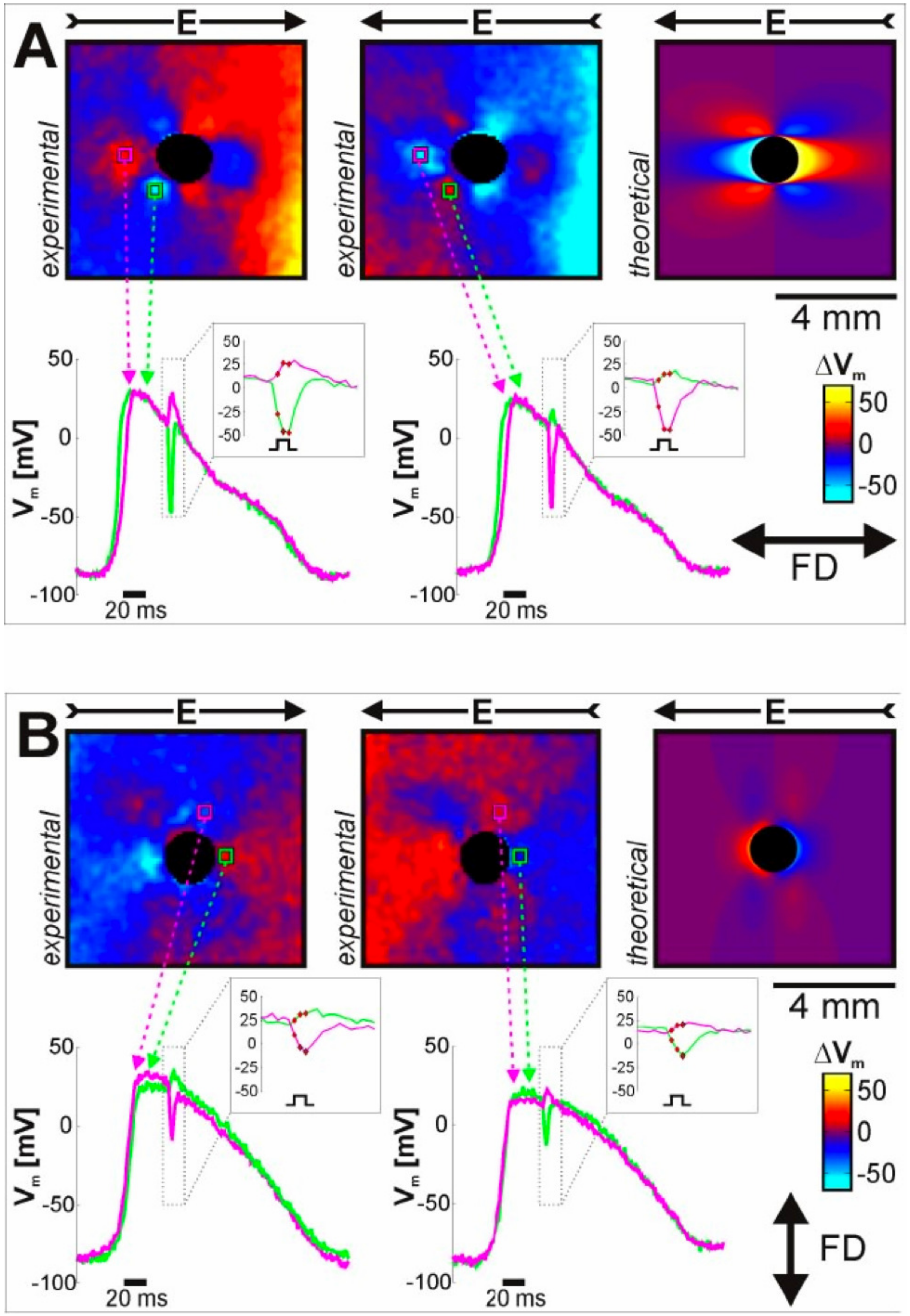
Woods et al., “Virtual electrode effects around an artificial heterogeneity during field stimulation of cardiac tissue” Heart Rhythm, 3:751-751, 2006



y
x
J_e
J_i
D
J_i
H
J_e
J_e
J_i



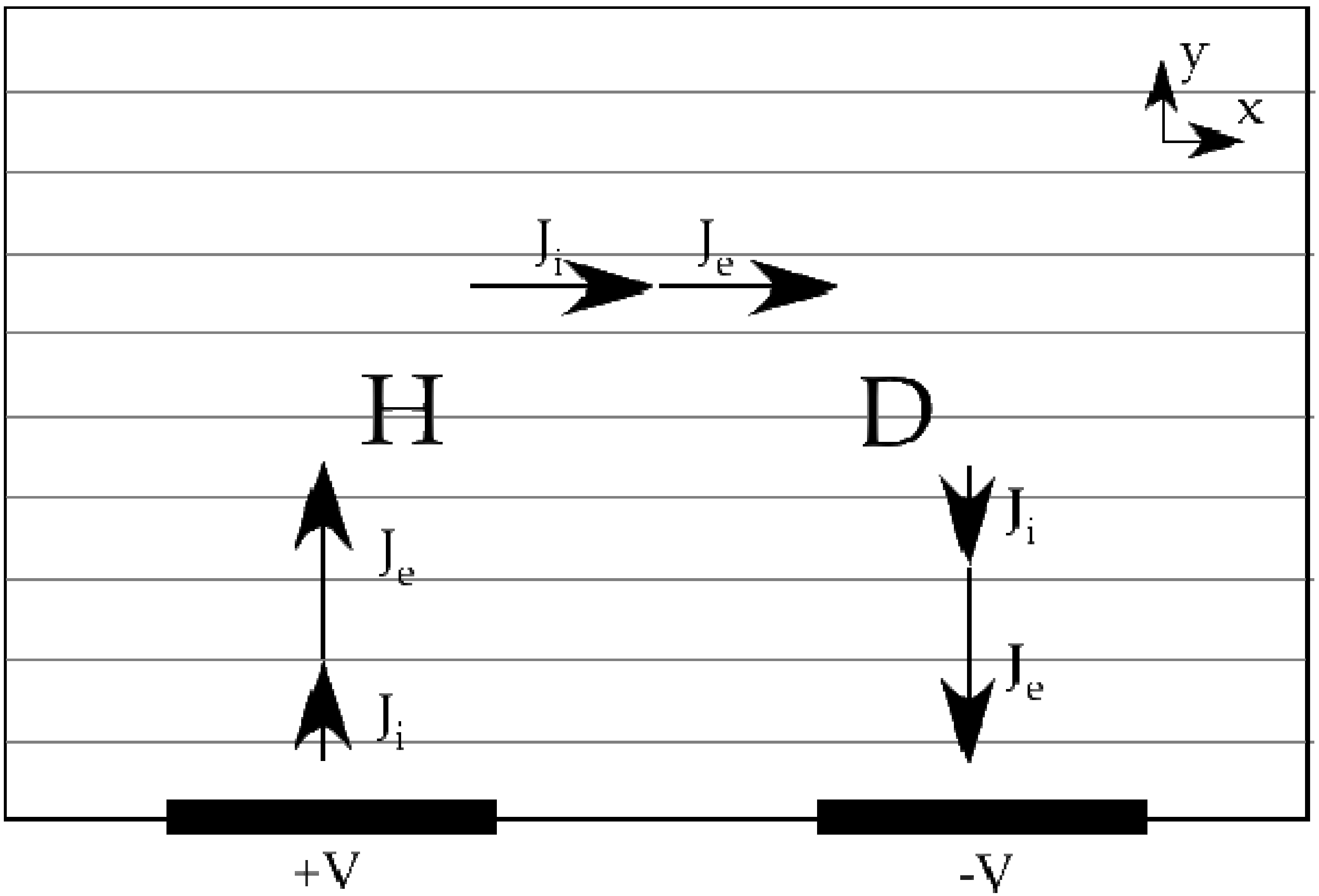
Woods et al., “Virtual electrode effects around an artificial heterogeneity during field stimulation of cardiac tissue” Heart Rhythm, 3:751-751, 2006



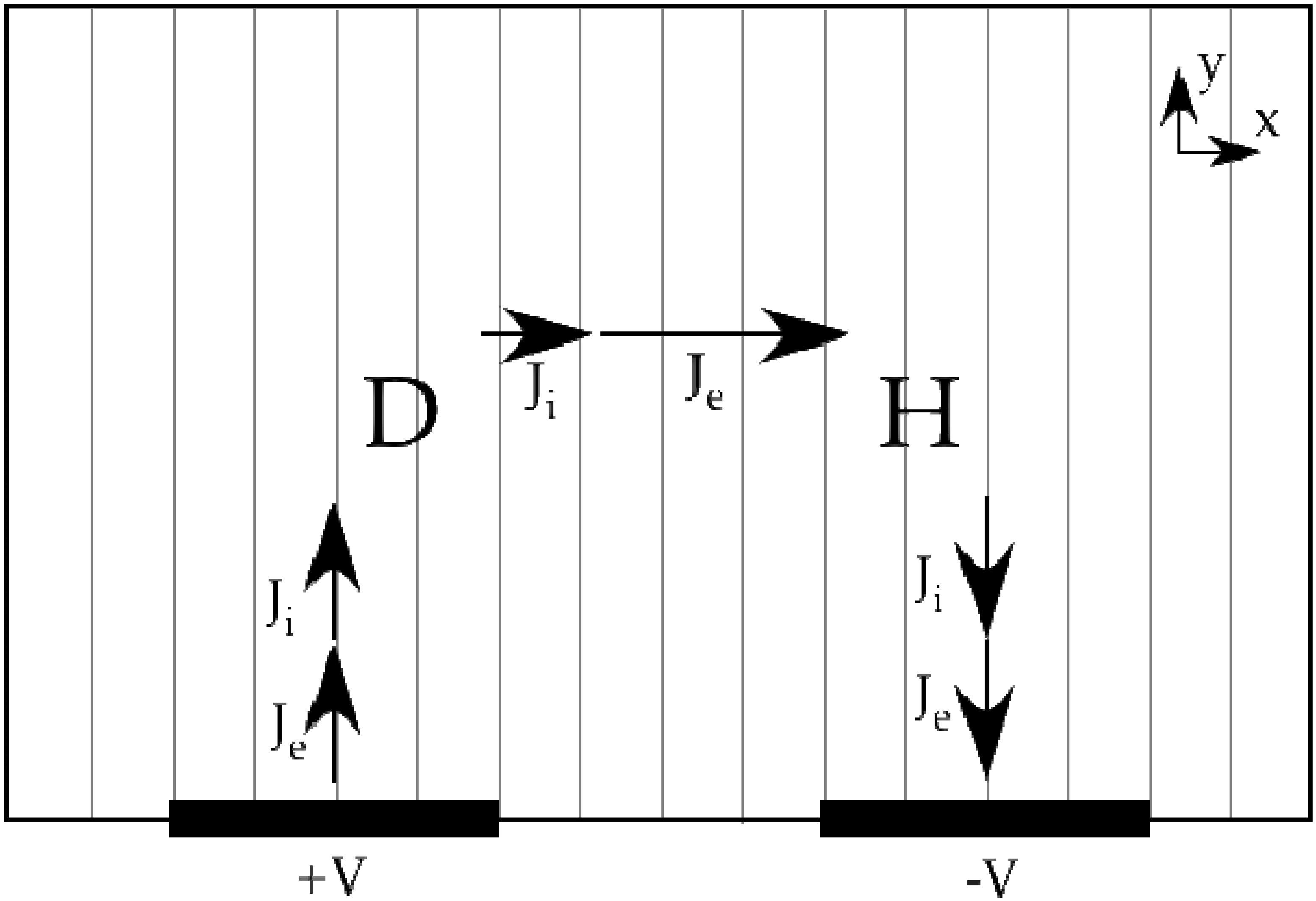
A
E
experimental
theoretical
4 mm
ΔVm
50
0
-50
FD
Vm [mV]
20 ms
B

Example 4: The transmembrane potential

induced by a inhomogeneous electric field

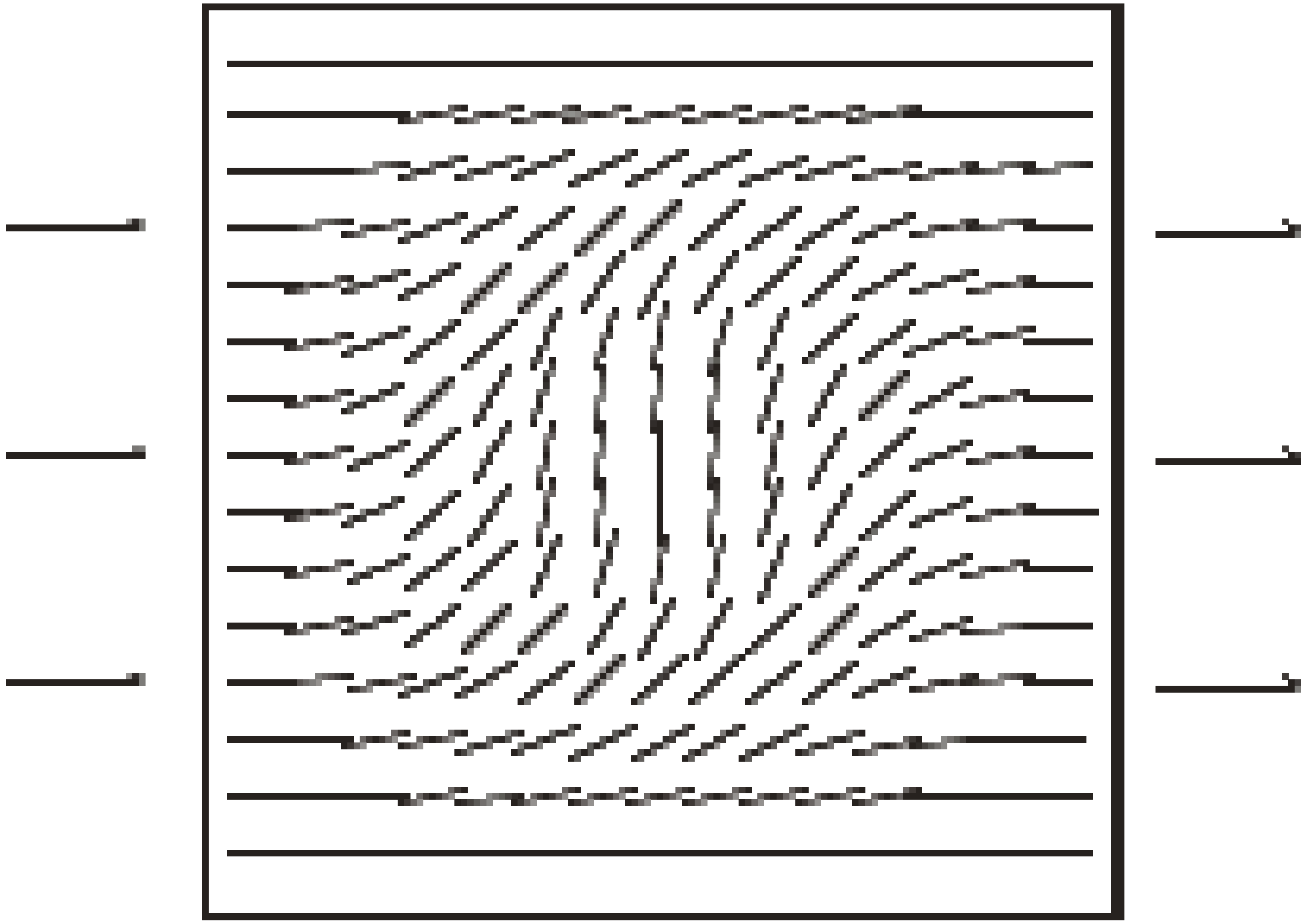


Otani, "Deep entry of defibrillating effects into homogeneous cardiac tissue" IEEE Trans. Biomed. Eng., 51:401-407, 2004

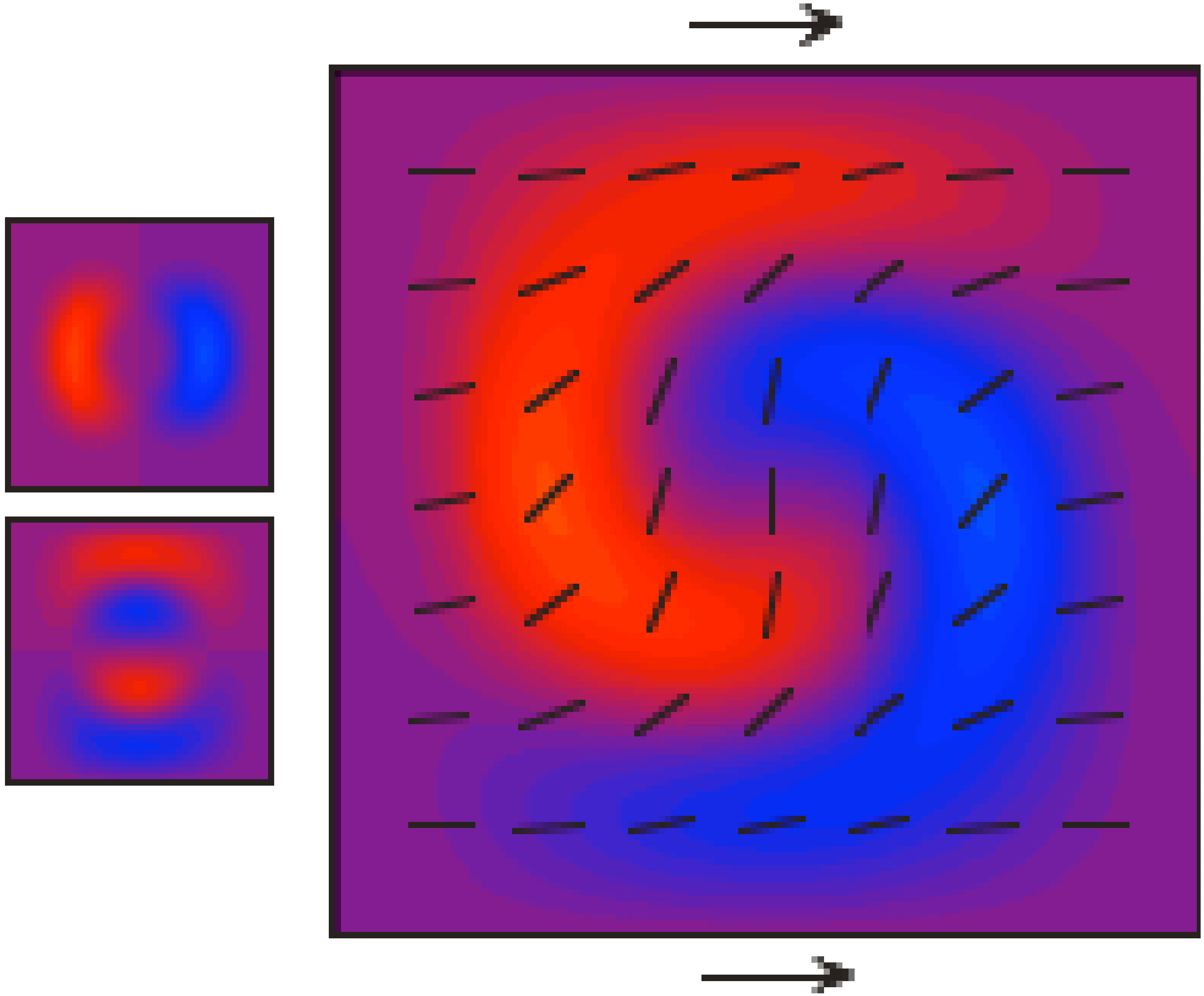


y
x
J_i
J_e
D
H
J_i
J_e
J_i
J_e
+V
-V

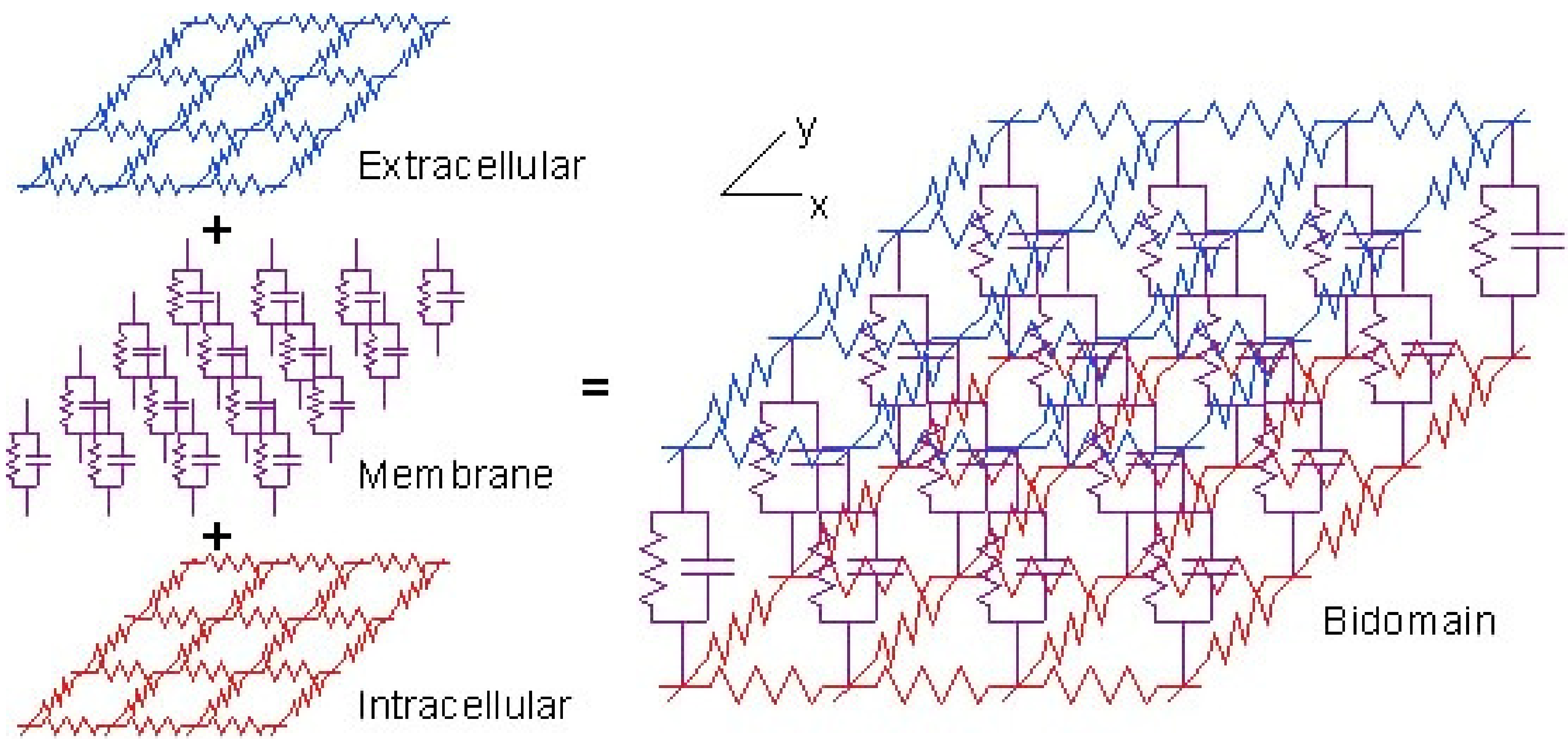
Example 5: Fiber curvature

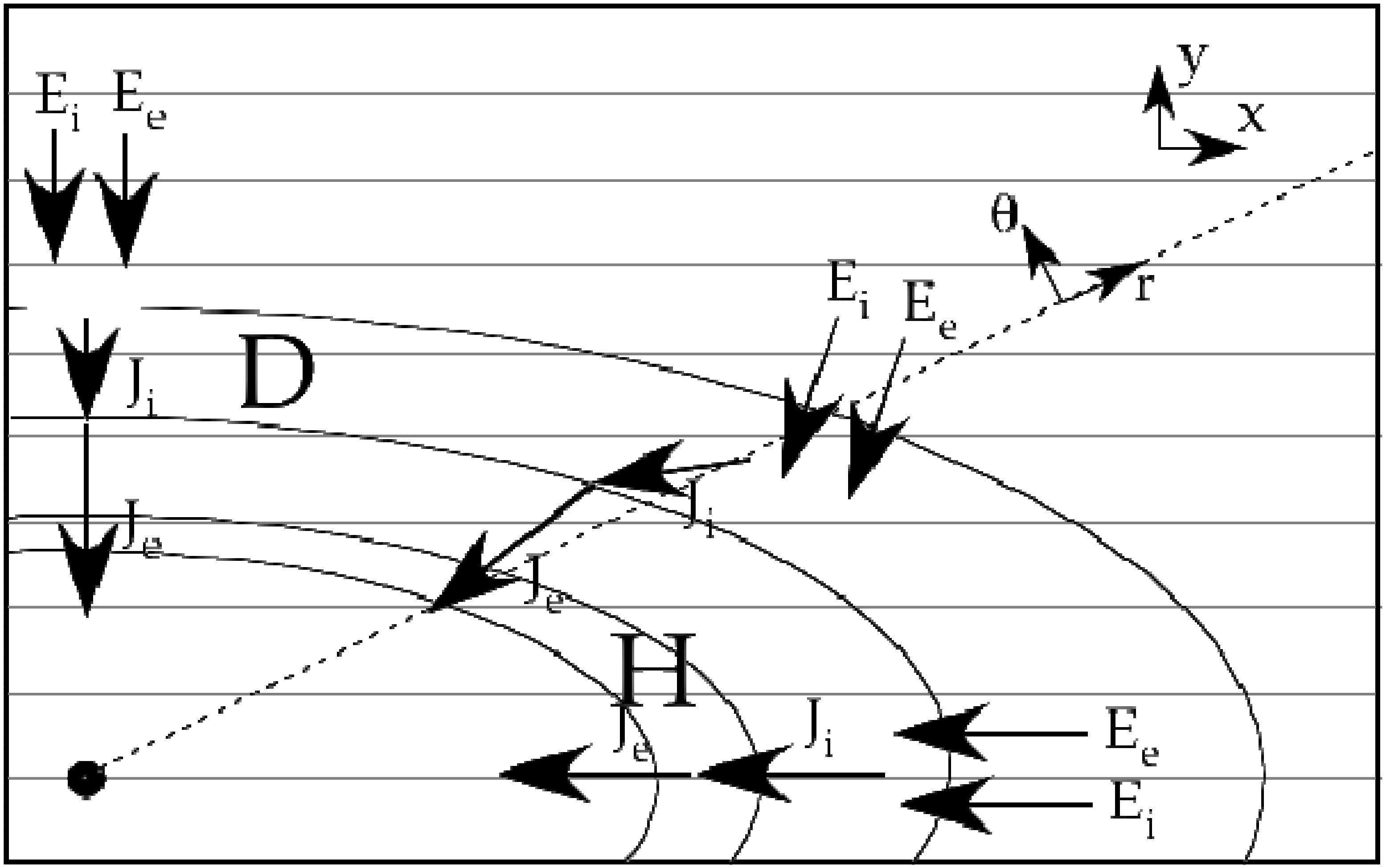


Roth and Langrill Beaudoin, “Approximate analytical solutions of the Bidomain equations for electrical stimulation of cardiac tissue with curving fibers” Phys. Rev. E, 67:051925, 2003

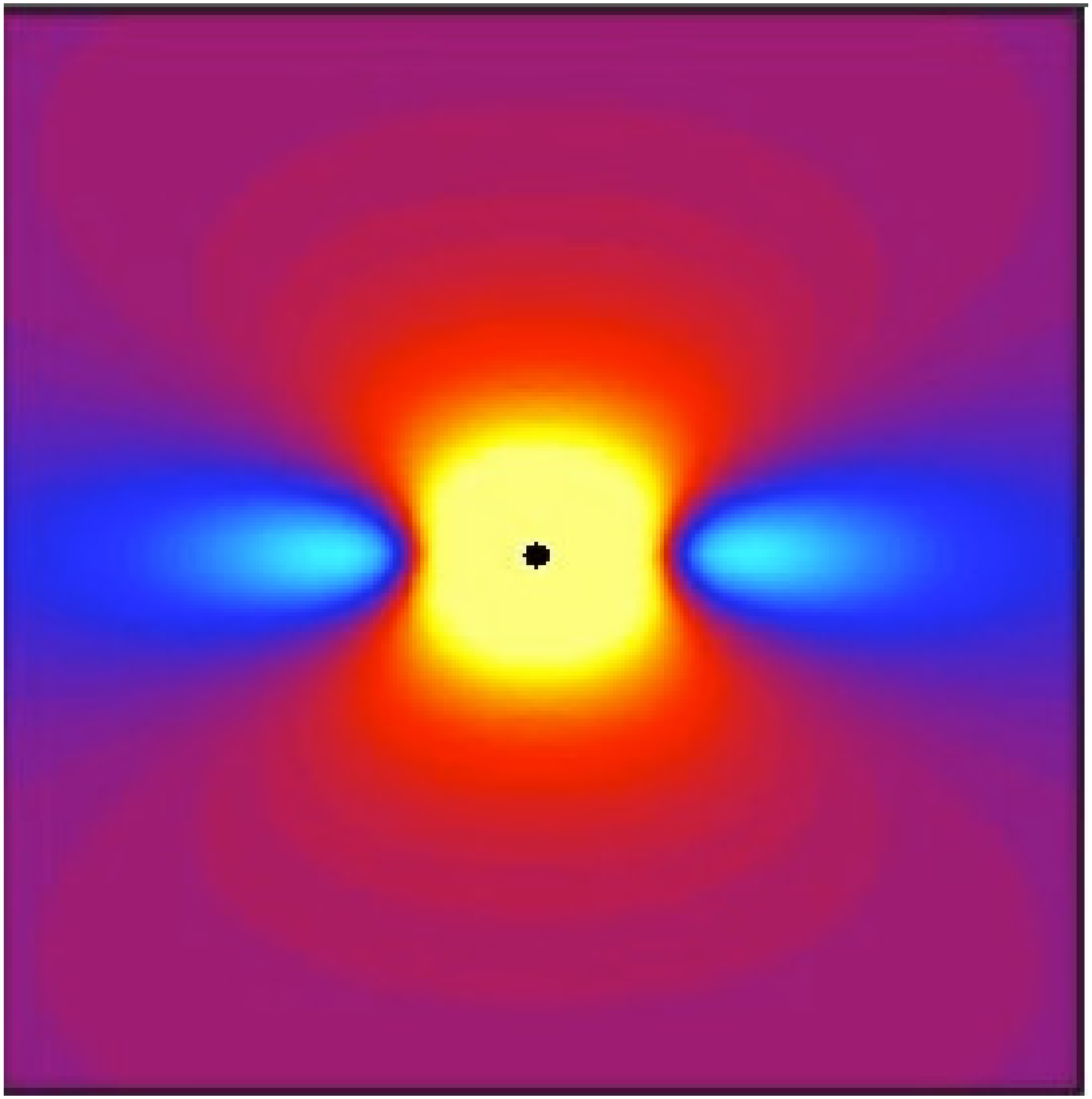


Example 6: The transmembrane potential induced by current injection through an extracellular unipolar electrode

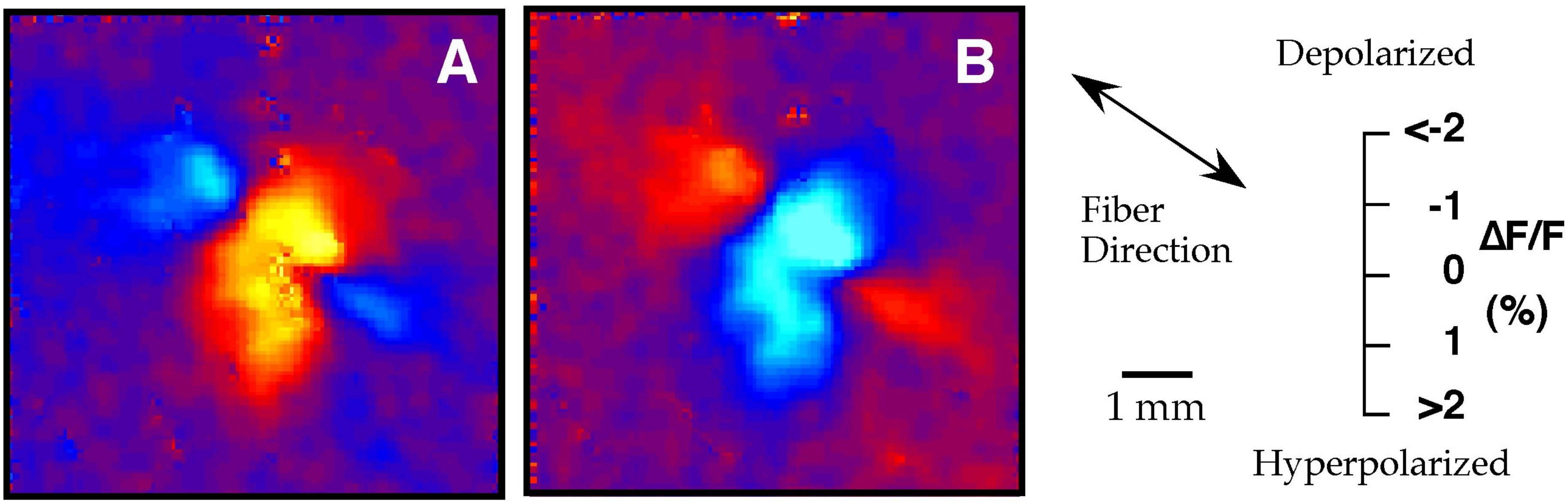

Extracellular
+
Membrane
+
Intracellular
=
y
x
Bidomain



y
x
E_i
E_e
θ
r
J_i
D
J_e
H
E_e
E_i

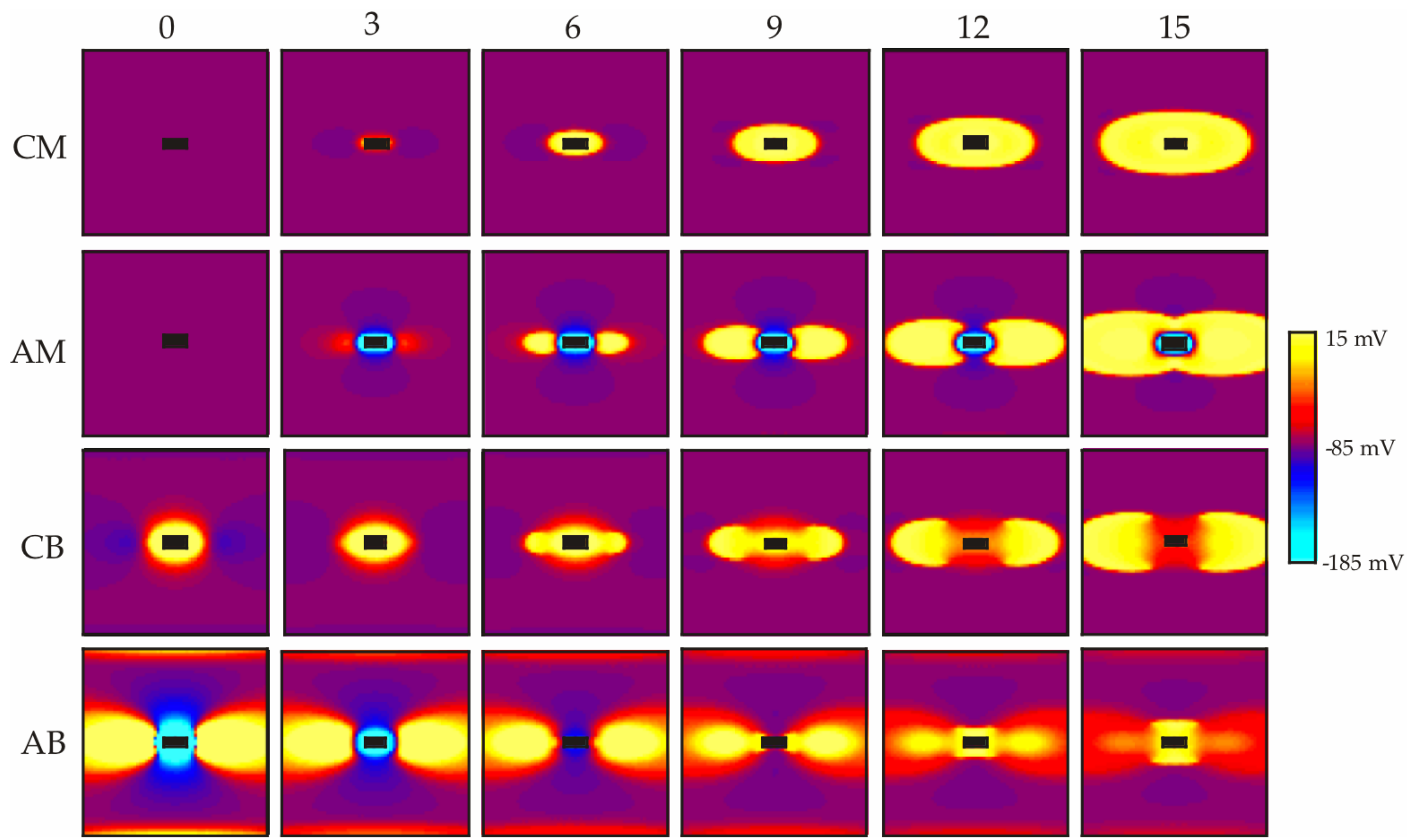


Sepulveda, Roth and Wikswo, “Current injection into a two-dimensional anisotropic bidomain” Biophys. J., 55:987-999, 1989

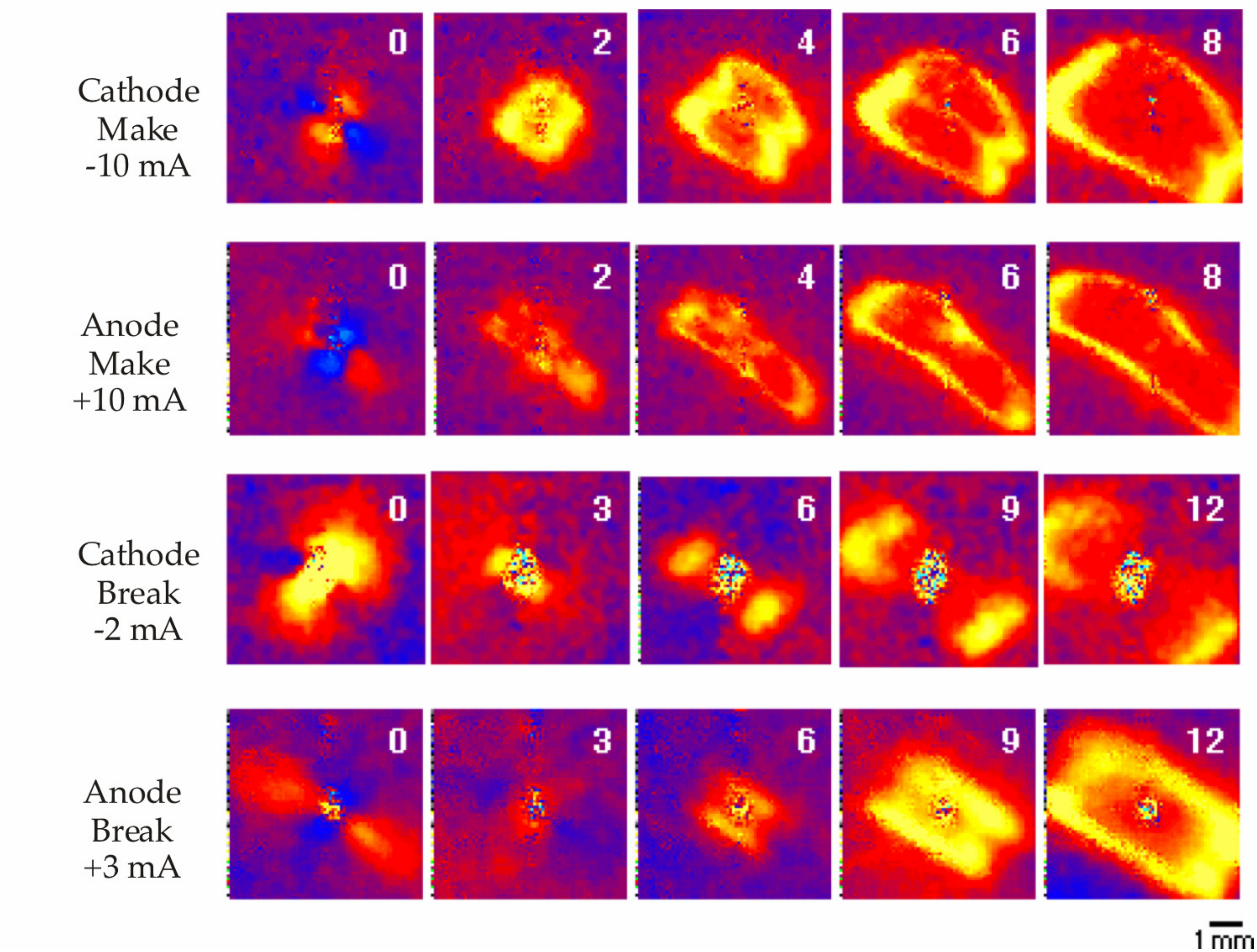


Wikswo, Lin and Abbas, “Virtual electrodes in cardiac tissue: A common mechanism for anodal and cathodal stimulation” Biophys. J., 69:2195-2210, 1995

Example 7: Four mechanisms of excitation

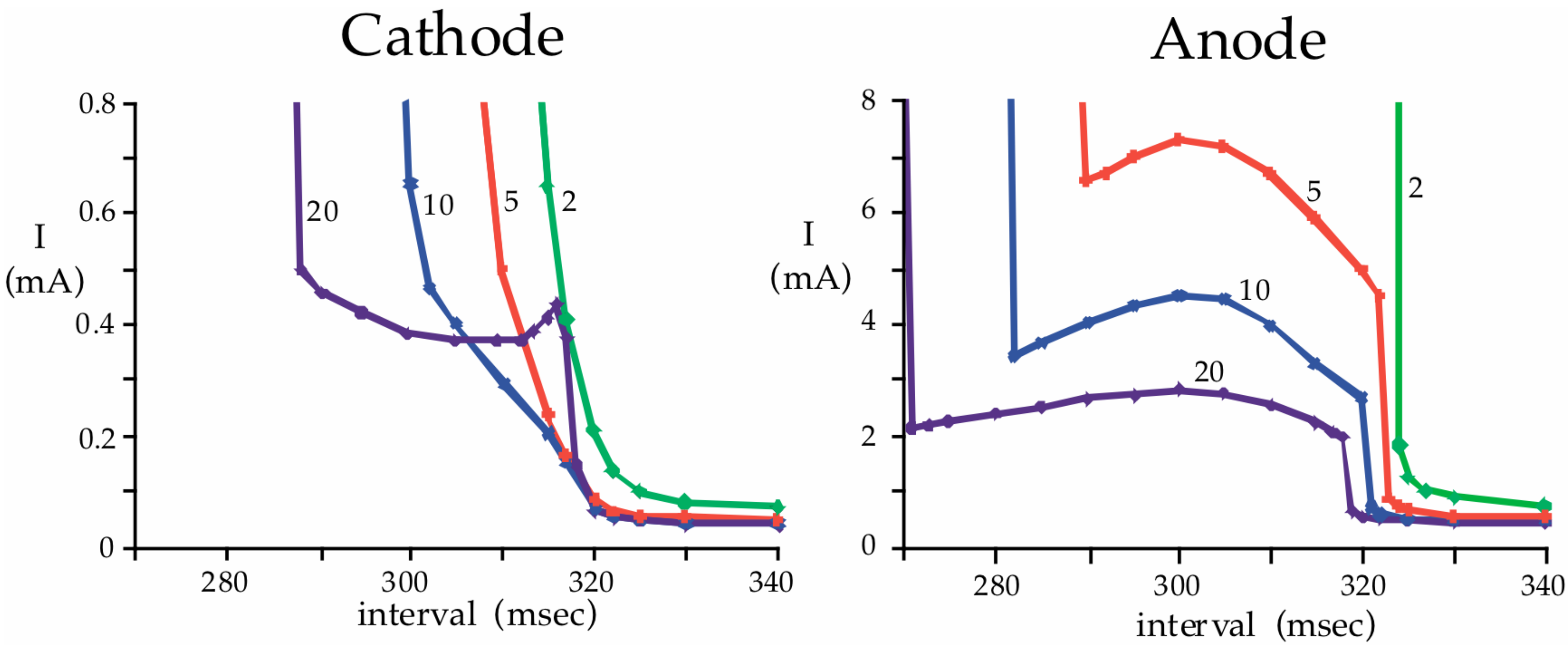


Roth, “A mathematical model of make and break electrical stimulation of cardiac tissue by a unipolar anode or cathode” IEEE Trans. Biomed. Eng., 42:1174-1184, 1995

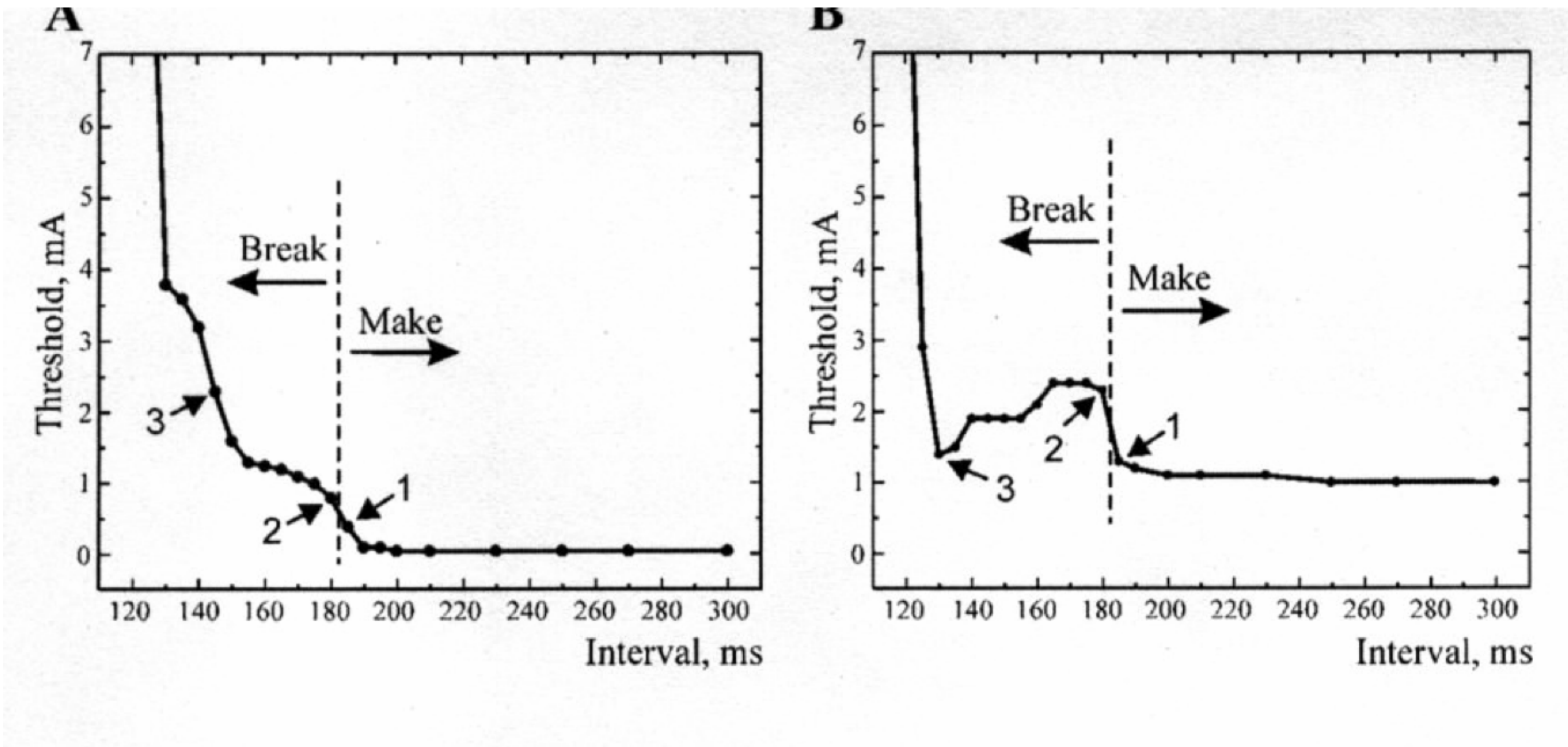


Wikswo, Lin and Abbas, "Virtual electrodes in cardiac tissue: A common mechanism for anodal and cathodal stimulation" Biophys. J., 69:2195-2210, 1995

Example 8: The dip in the anodal strength-interval curve

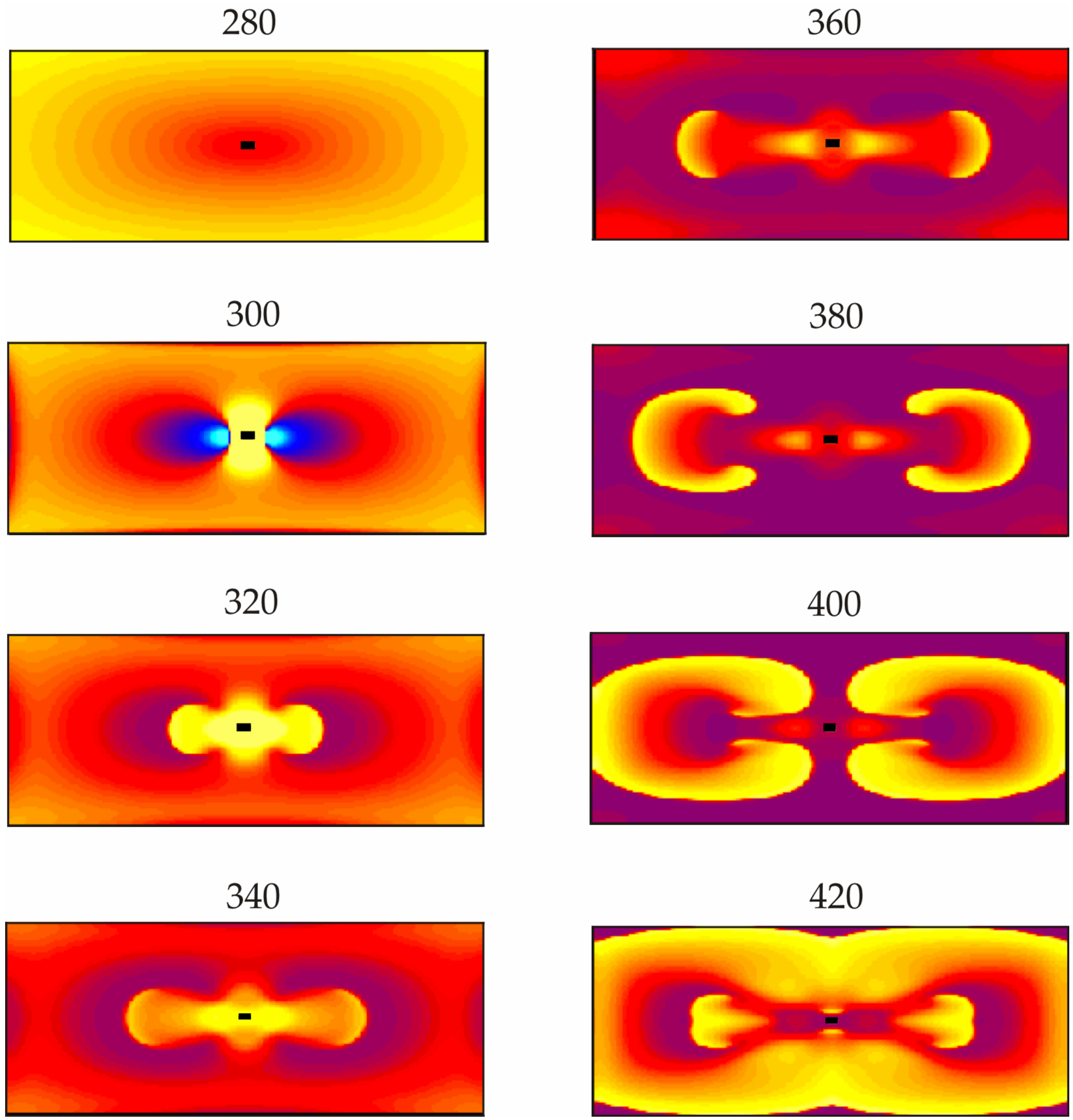


Roth, “Strength-interval curves for cardiac tissue predicted using the bidomain model” J. Cardiovasc. Electrophysiol., 7:722-737, 1996

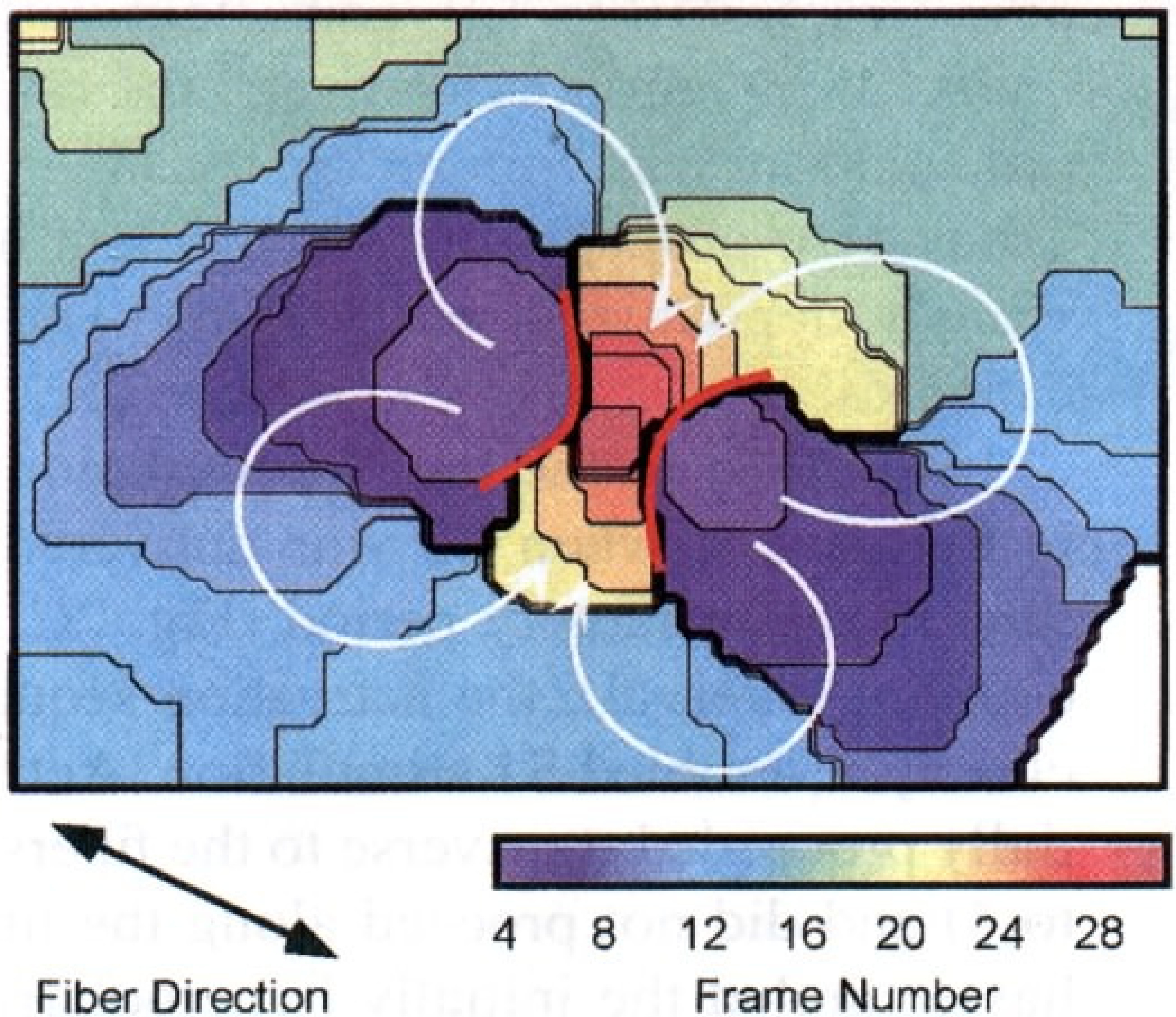


Sidorov, Woods, Baudenbacher and Baudenbacher, “Examination of stimulation mechanism and strength-interval curve in cardiac tissue” Amer. J. Physiol., 289:H2602–H2615, 2005

Example 9: Quatrefoil reentry

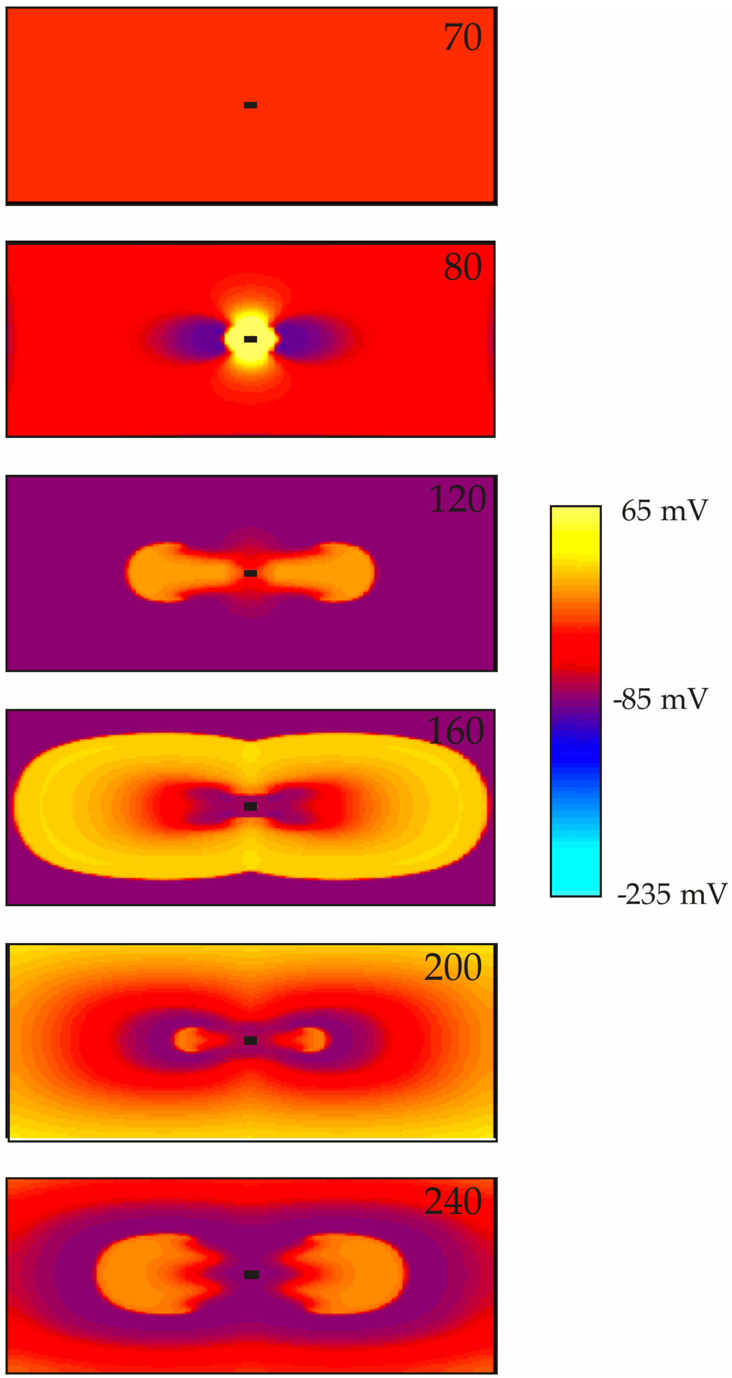


Roth, “Nonsustained reentry following successive stimulation of cardiac tissue through a unipolar electrode” J. Cardiovasc. Electrophysiol., 8:768-778, 1997

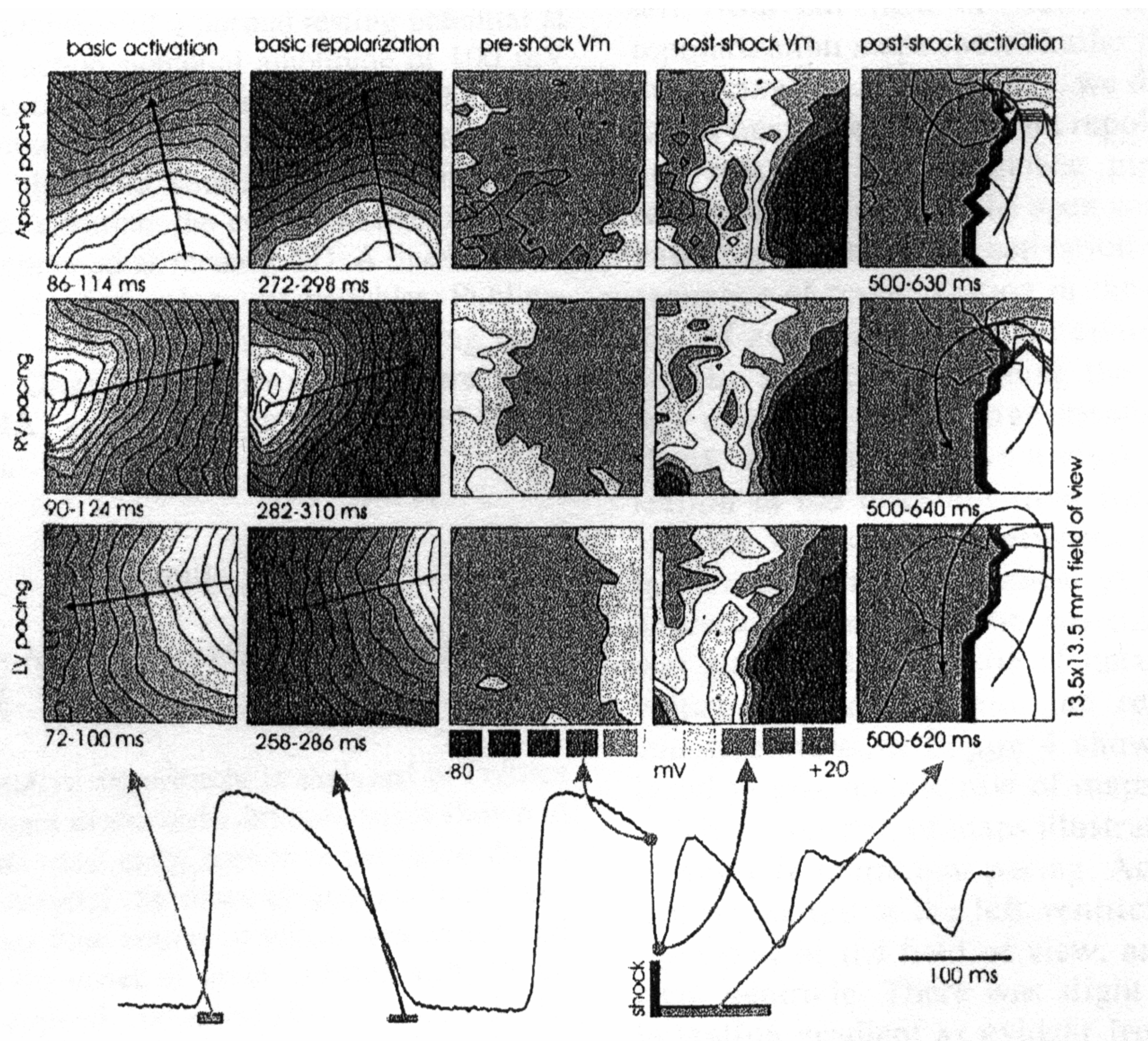


Lin, Roth and Wikswo, "Quatrefoil reentry in myocardium: An optical imaging study of the induction mechanism" J. Cardiovasc. Electrophysiol., 10:574-586, 1999

Example 10: An S1 refractory gradient is not necessary for S2 reentry

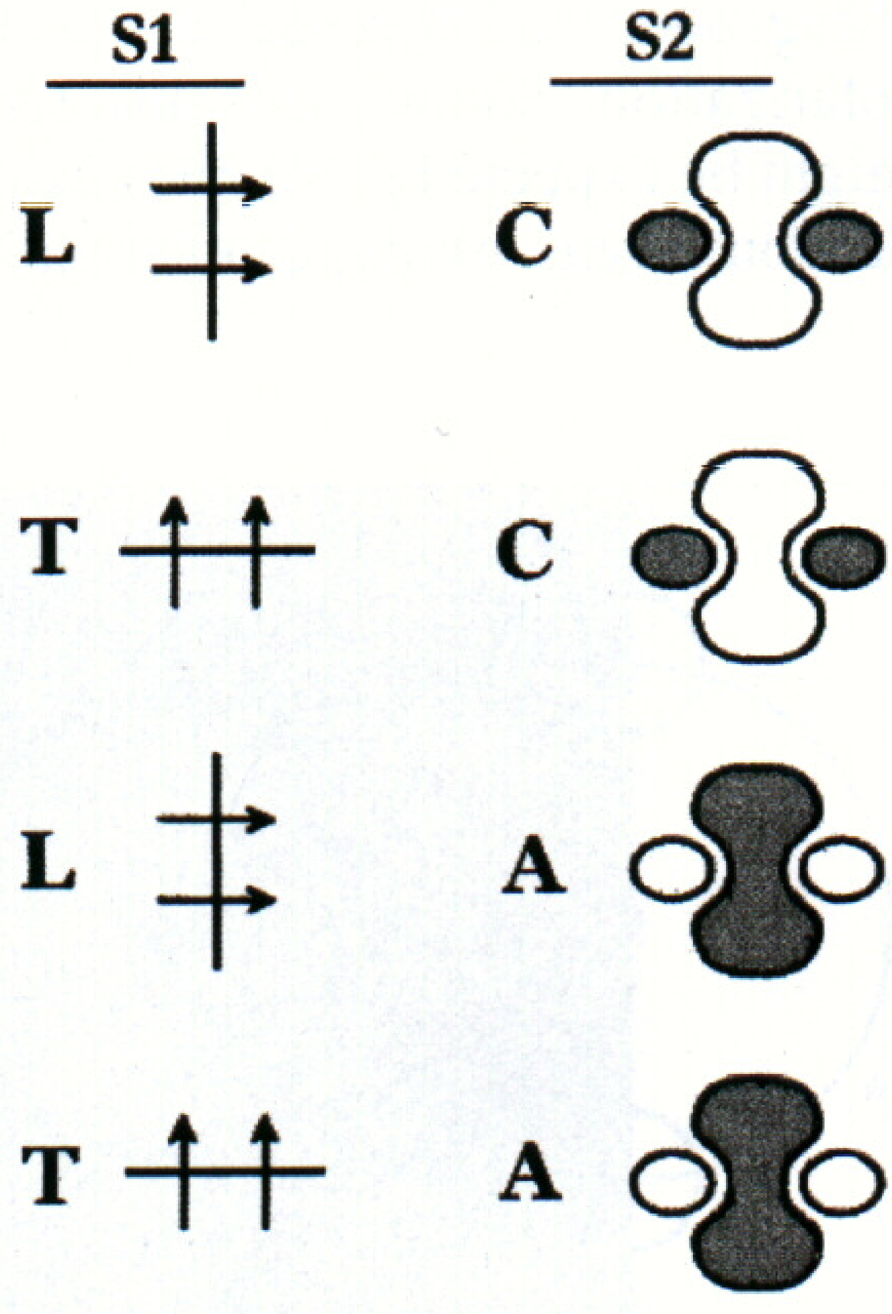


Roth, “An S1 gradient of refractoriness is not essential for reentry induction by an S2 stimulus” IEEE Trans. Biomed. Eng., 47:820-821, 2000

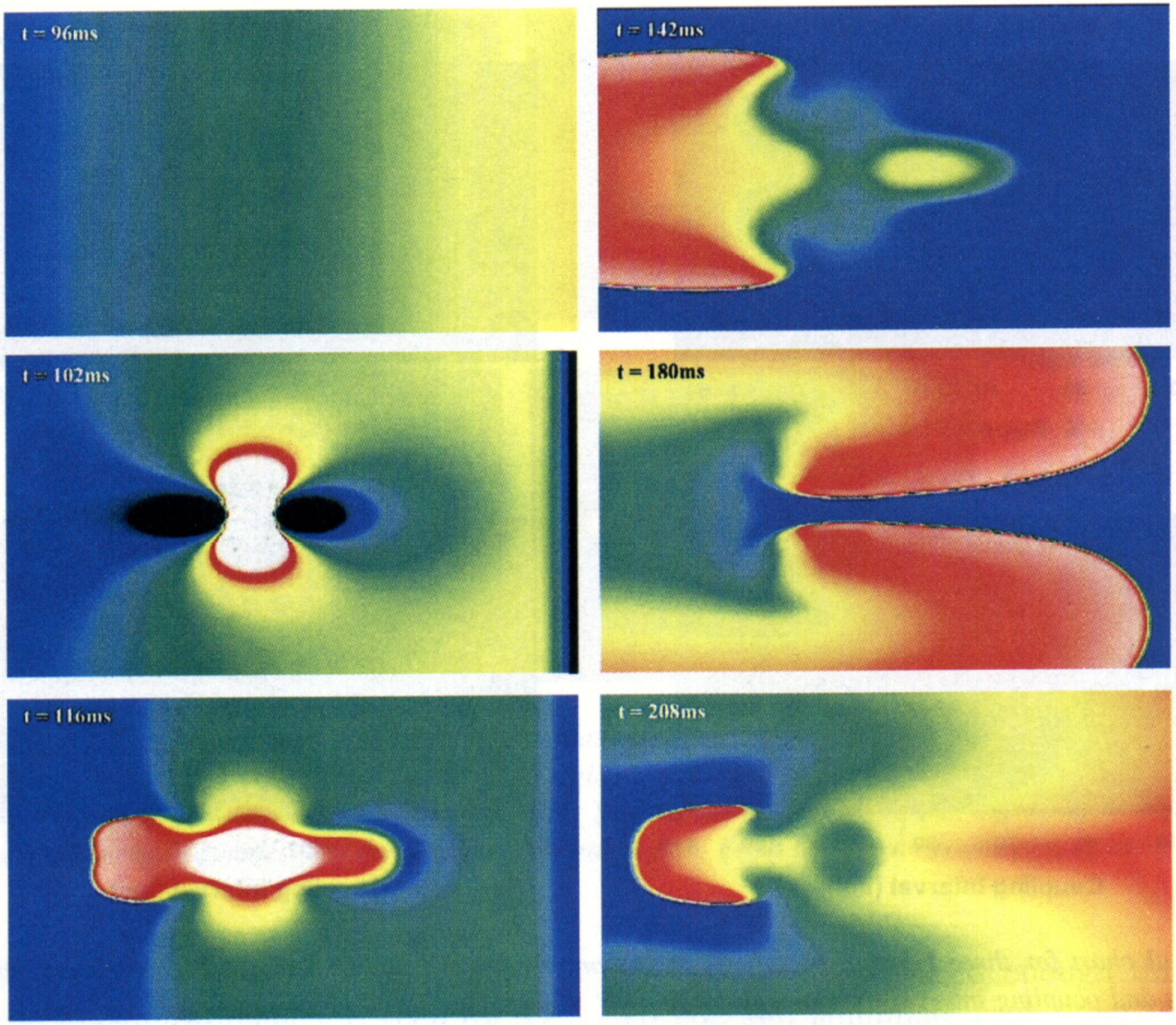


Cheng, Nikolski and Efimov, "Reversal of repolarization gradient does not reverse the chirality of shock-induced reentry in the rabbit heart" J. Cardiovasc. Electrophysiol., 11:998-1007, 2000

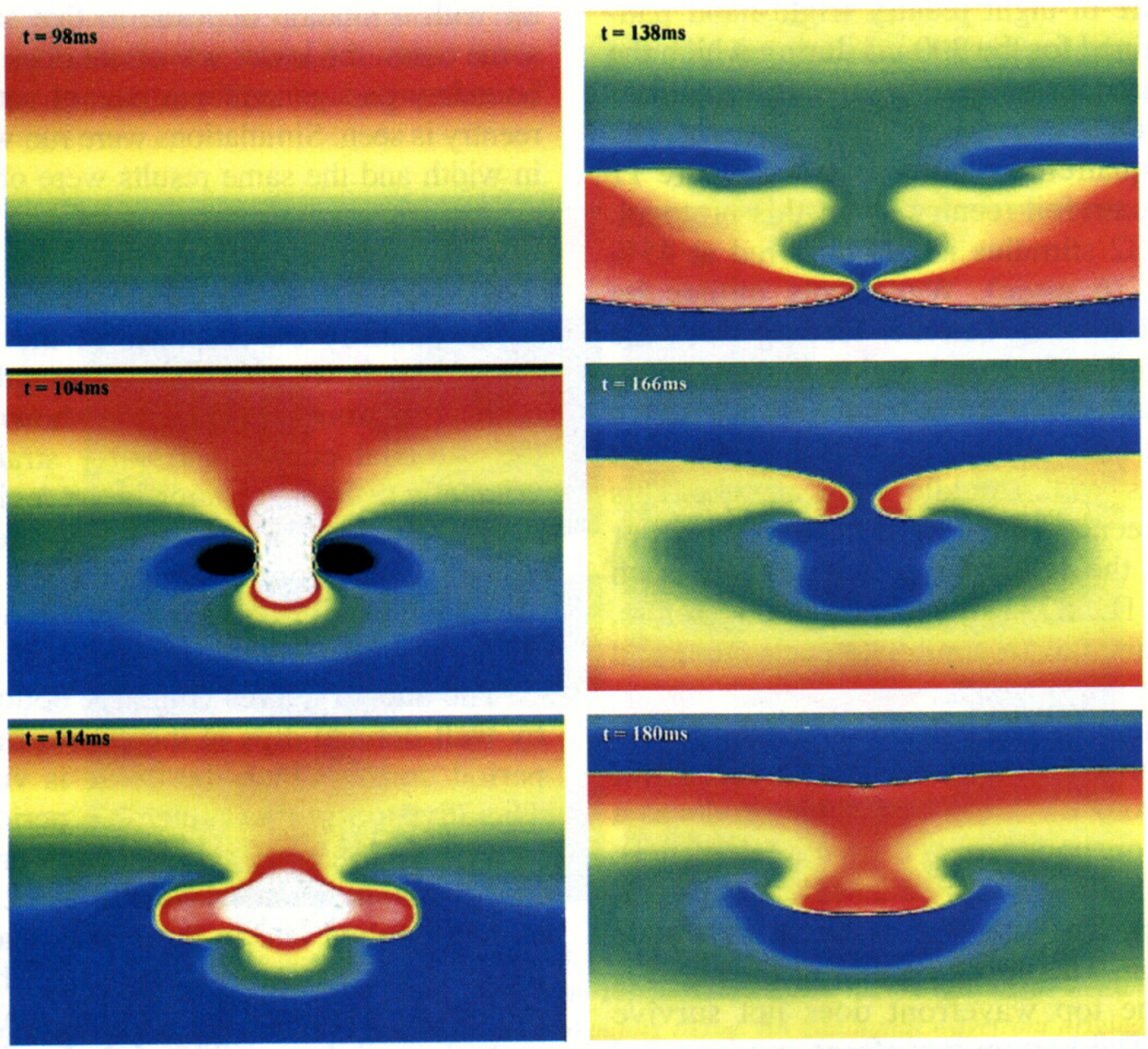
Example 11: The pinwheel experiment revisited



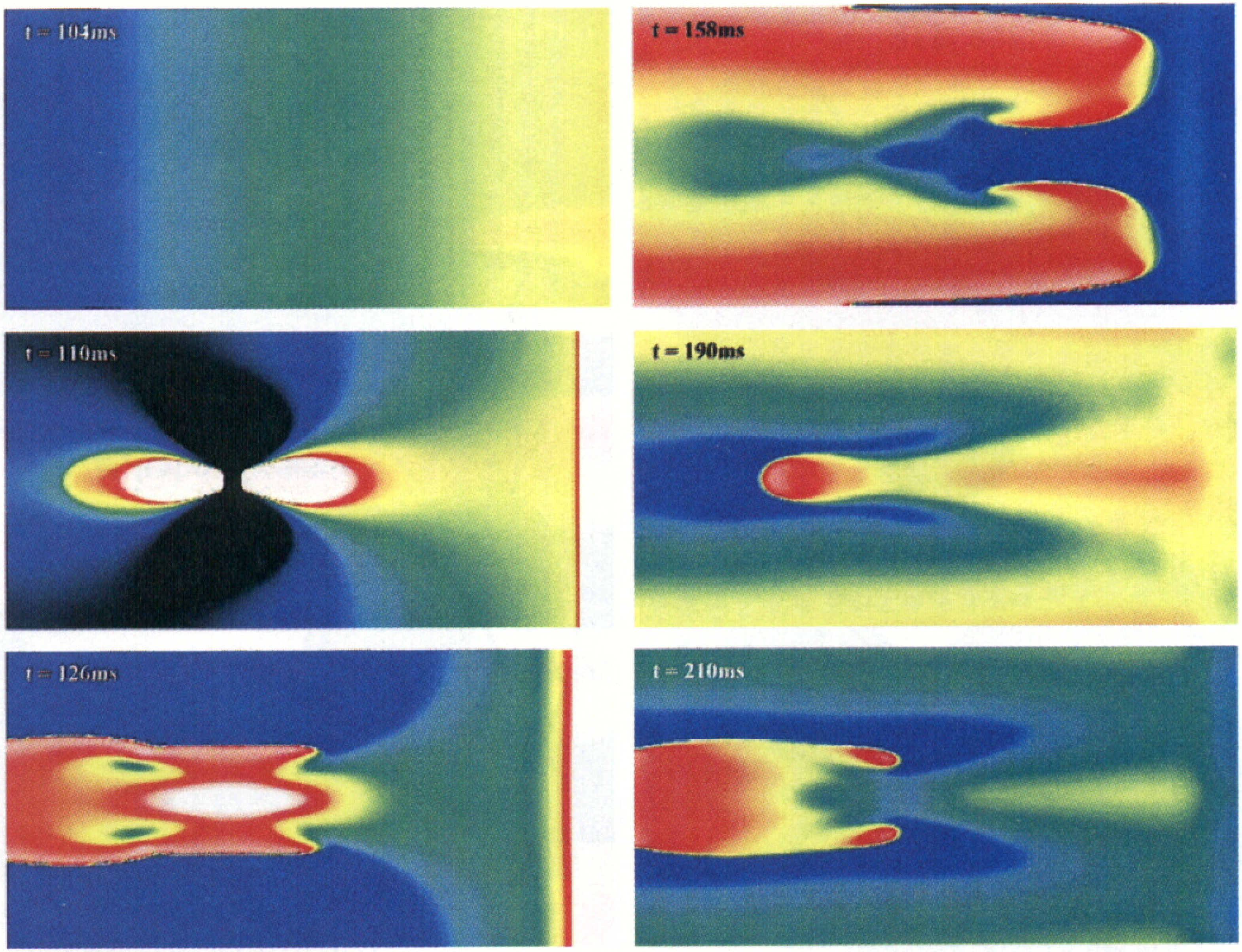
S1
S2
L
C
T
C
L
A
T
A



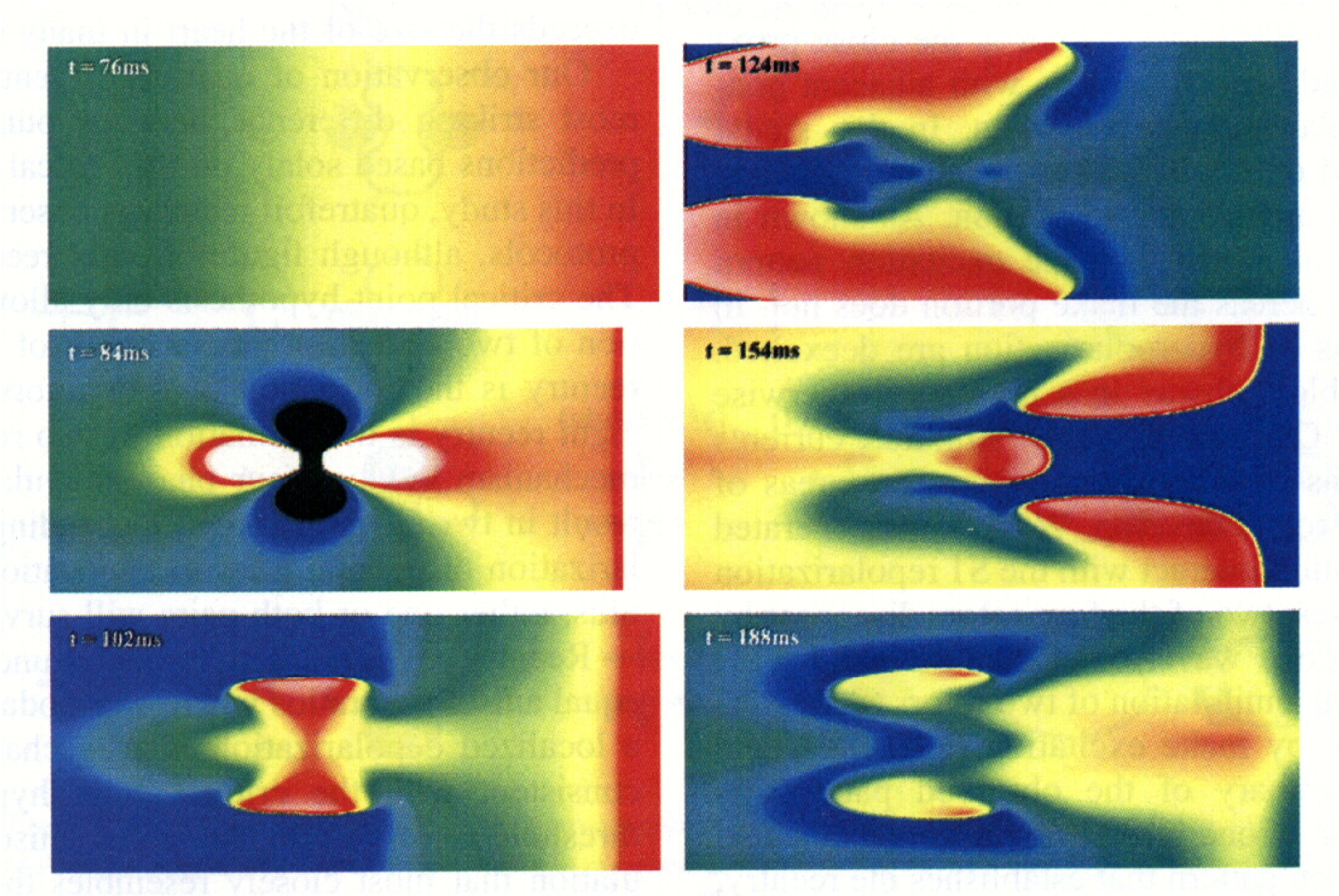
S1L S2C



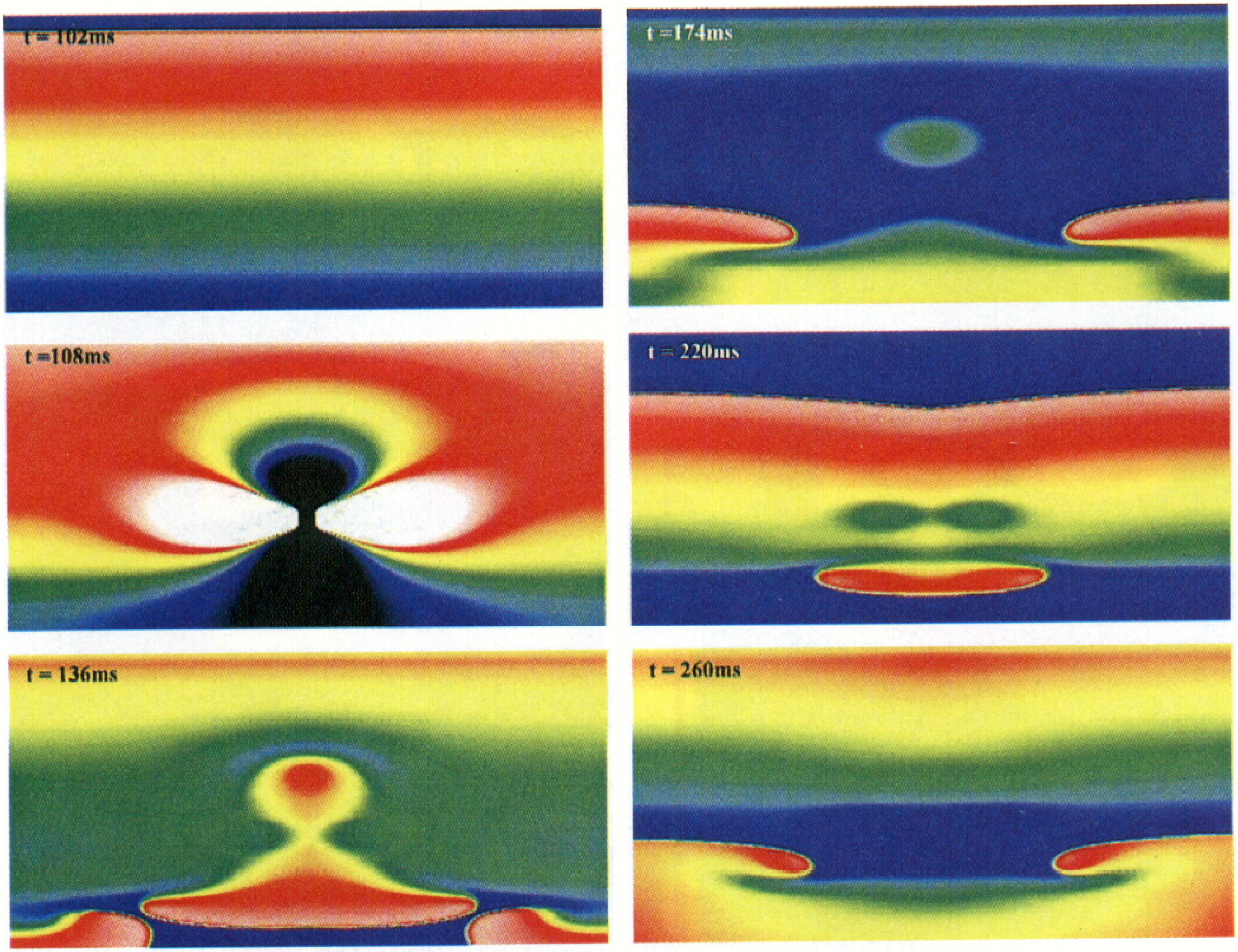
S1T S2C



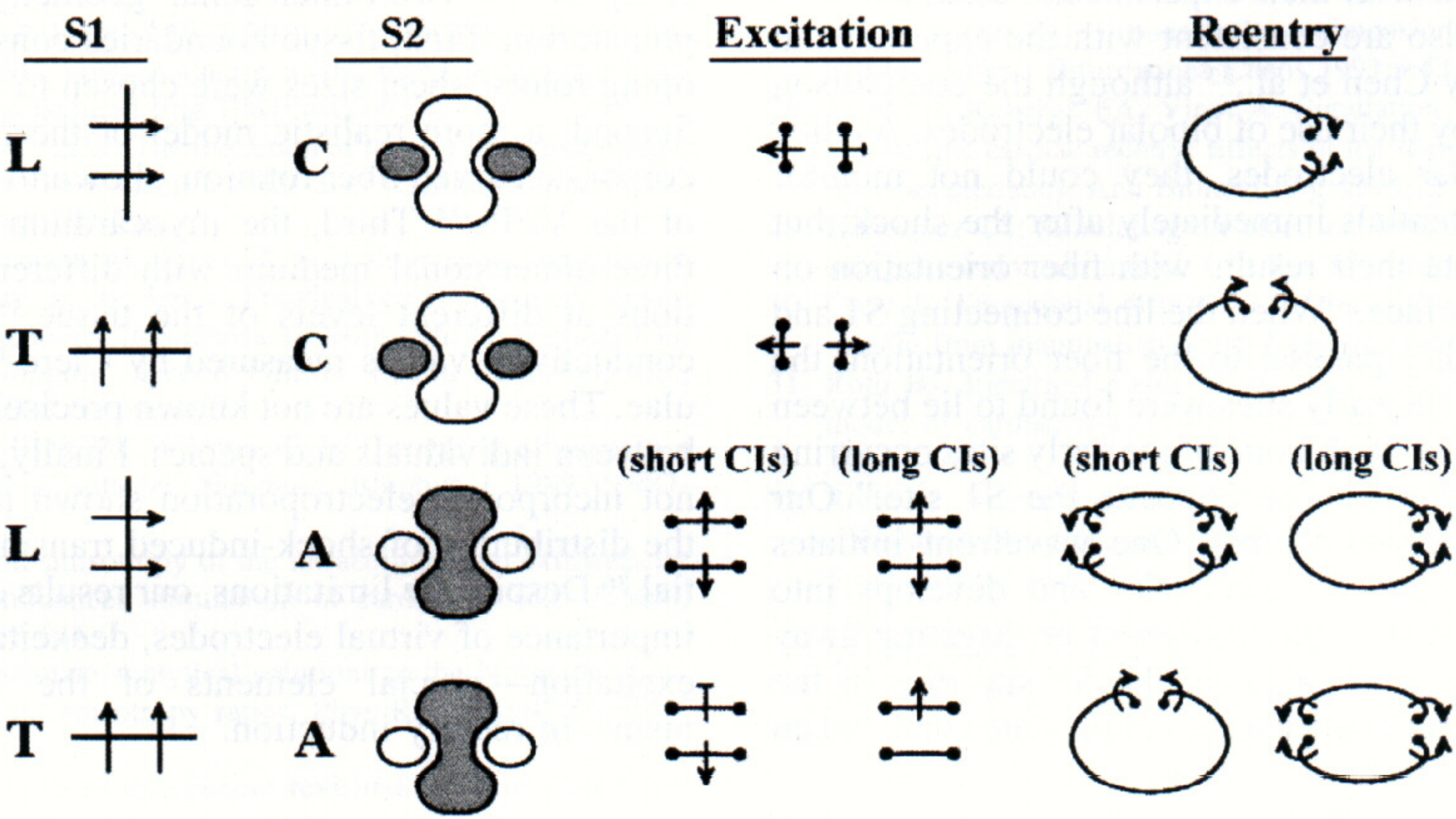
S1L S2A

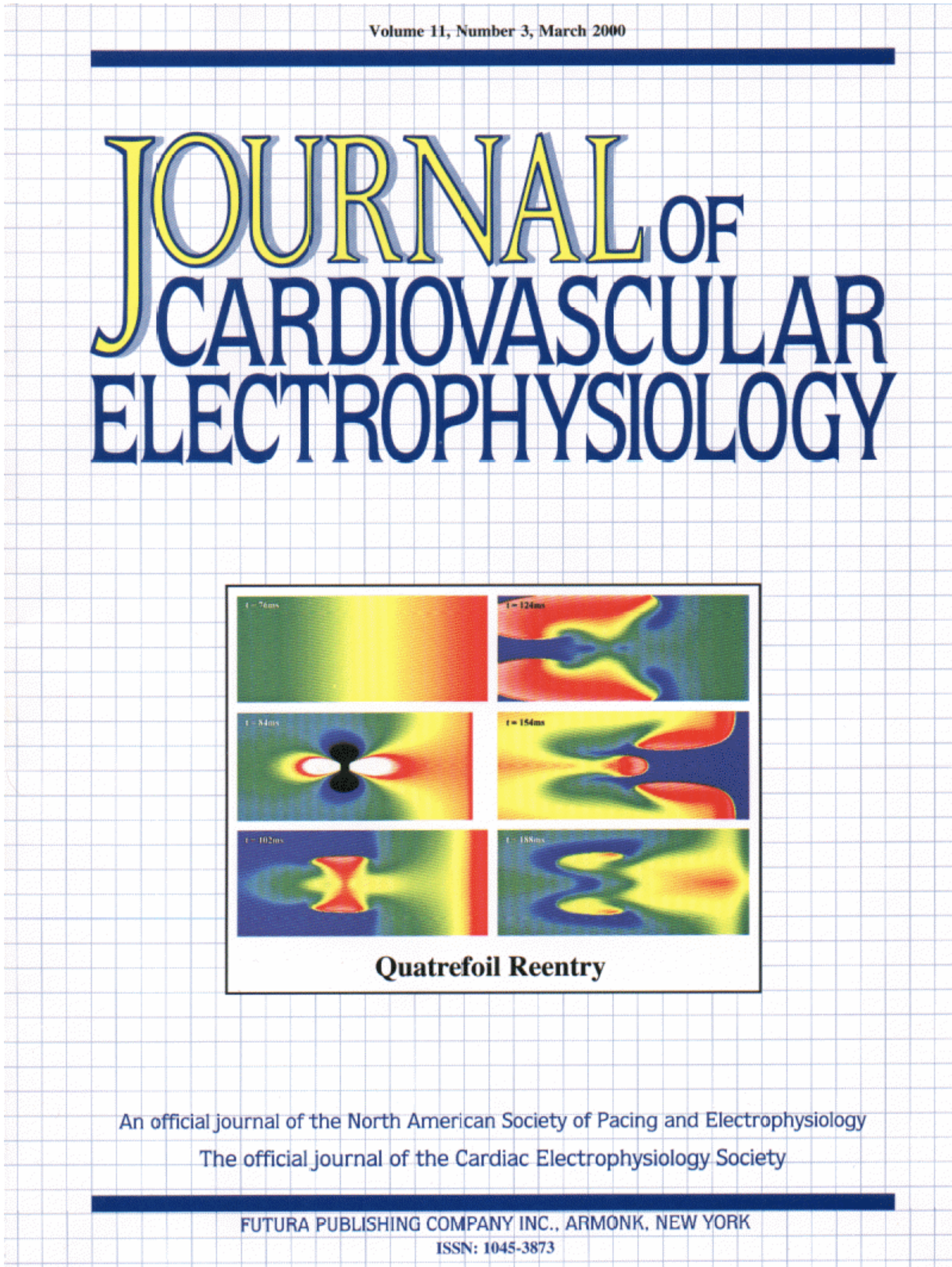


S1L S2A



S1T S2A


S1
S2
Excitation
Reentry
L
C
T
C
(short CIs)
(long CIs)
(short CIs)
(long CIs)
L
A
T
A



Lindblom, Roth and Trayanova, "Role of Virtual Electrodes in Arrhythmogenesis: The Pinwheel Experiment Revisited" J. Cardiovasc. Electrophsyiol. 11:274-285, 2000

Various Ways to Make Phase Singularities by Electric Shock

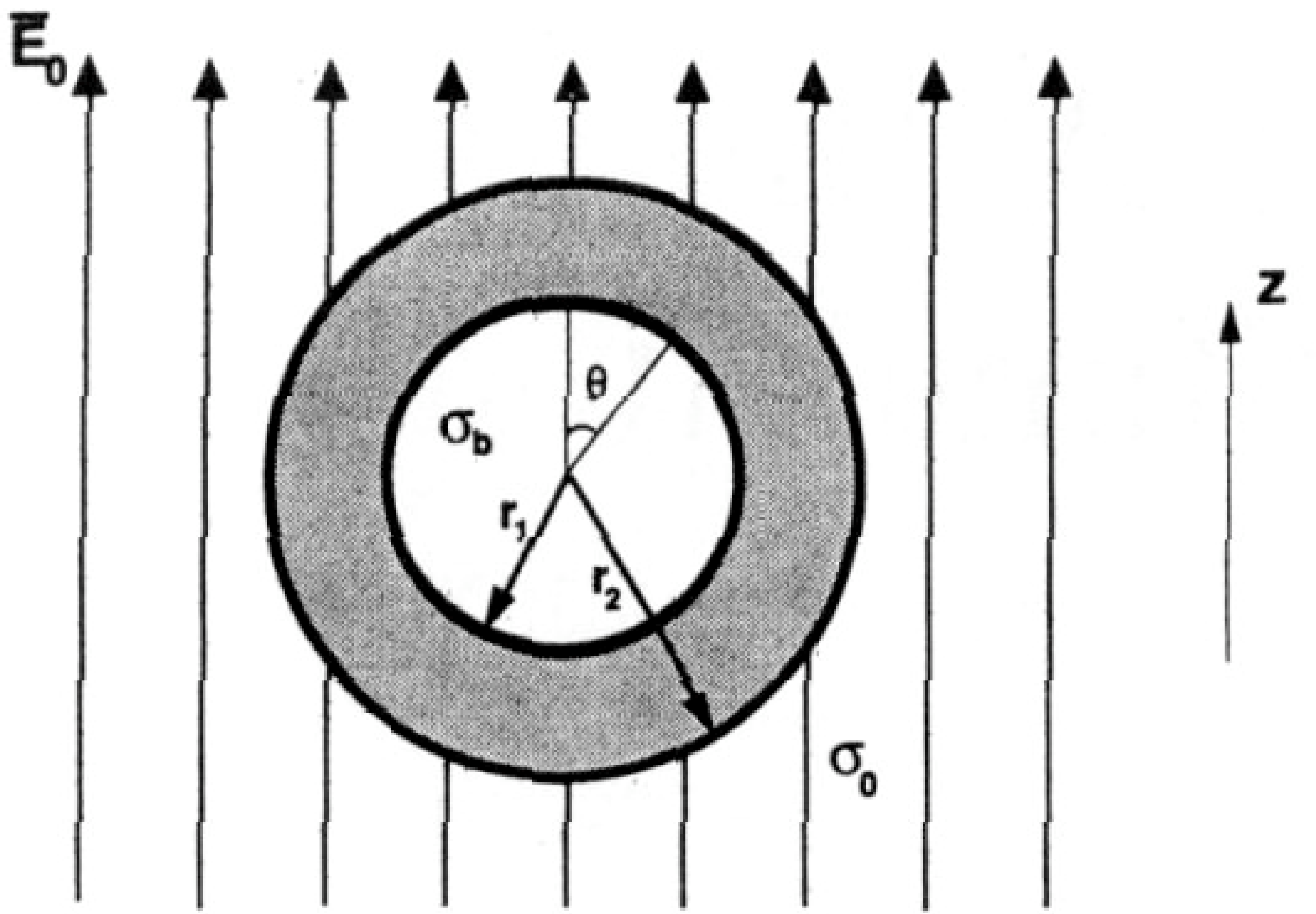
ARTHUR T. WINFREE, PH.D.
From the Department of Ecology and Evolutionary Biology, University of Arizona, Tucson, Arizona

"This is clearly a landmark event in cardiac electrophysiology at the end of our century. It is sure to have major implications for clinical electrophysiologic work and for defirbillator design."

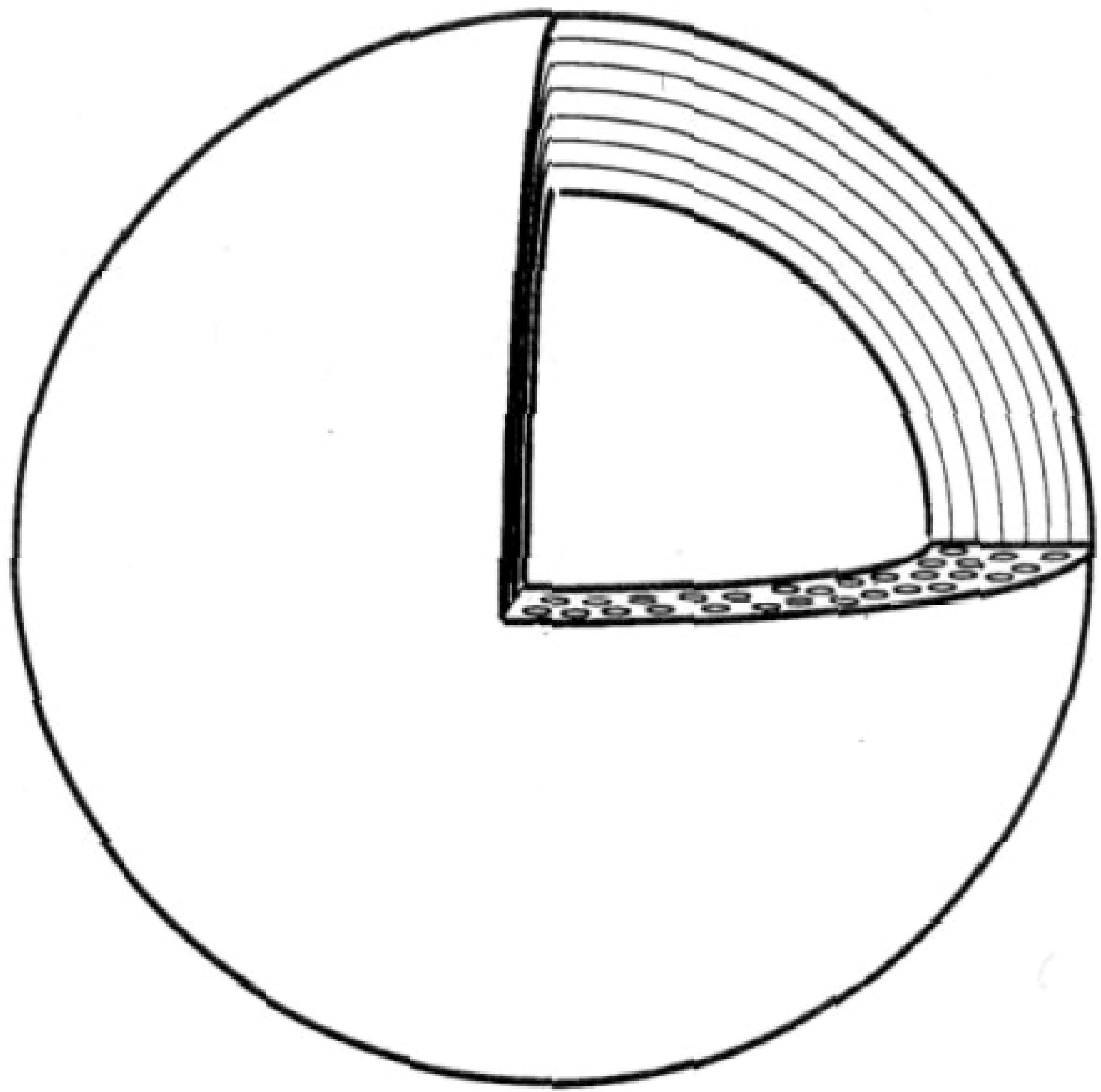
Journal of Cardiovascular Electrophysiology

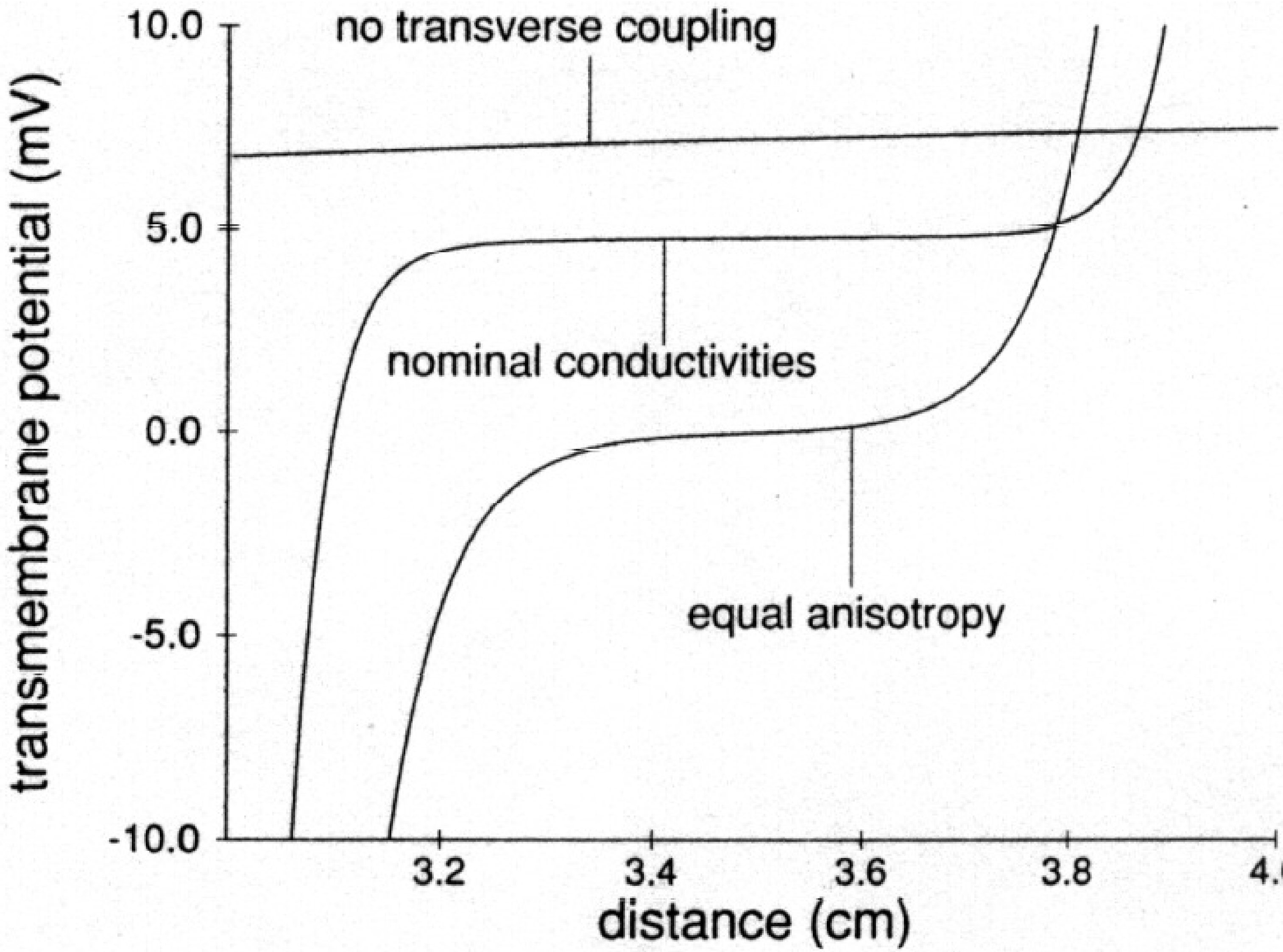
March 2000

Example 12: Spherical heart and defibrillation



Trayanova, Roth and Malden, “The response of a spherical heart to a uniform electric field: A bidomain analysis of cardiac stimulation” IEEE Trans. Biomed. Eng., 40:899-908,




no transverse coupling
nominal conductivities
equal anisotropy
10.0
5.0
0.0
-5.0
-10.0
transmembrane potential (mV)
3.2
3.4
3.6
3.8
4.0
distance (cm)

(d)

Examples

1. The magnetic field of an action potential wave front
2. The transmembrane potential induced near an insulating boundary
3. The transmembrane potential induced around a circular insulator
4. The transmembrane potential induced by an inhomogeneous electric field
5. Fiber curvature
6. The transmembrane potential induced by unipolar current injection
7. Four mechanisms of excitation
8. The dip in the anodal strength-interval curve
9. Quatrefoil reentry
10. An S1 refractory gradient is not necessary for S2 reentry
11. The pinwheel experiment revisited
12. The spherical heart and defibrillation

Supported by the National Institutes of Health
and the American Heart Association--Midwest Affiliate

Bidomain equations

$$\sigma_{ix}\frac{\partial^2 V_i}{\partial x^2}+\sigma_{iy}\frac{\partial^2 V_i}{\partial y^2}=\beta\left(C_m\frac{\partial V_m}{\partial t}+J_{ion}\right)$$

$$\sigma_{ex}\frac{\partial^2 V_e}{\partial x^2}+\sigma_{ey}\frac{\partial^2 V_e}{\partial y^2}=-\beta\left(C_m\frac{\partial V_m}{\partial t}+J_{ion}\right)$$

Bidomain equations (rewritten)

$$\sigma_{ex} \frac{\partial^2 V_e}{\partial x^2} + \sigma_{ey} \frac{\partial^2 V_e}{\partial y^2} = -\beta \left(C_m \frac{\partial V_m}{\partial t} + J_{ion} \right)$$

$$(\sigma_{ix} + \sigma_{ex}) \frac{\partial^2 V_e}{\partial x^2} + (\sigma_{iy} + \sigma_{ey}) \frac{\partial^2 V_e}{\partial y^2} =$$

$$- \sigma_{ix} \frac{\partial^2 V_m}{\partial x^2} - \sigma_{iy} \frac{\partial^2 V_m}{\partial y^2}$$