$t = 0$ t_1

$N(t=0) = N_0$ $N(t_1)$

$N(t_1) =$ die + reproduce + survive w/o doing anything

\# of bugs at time t_1

$= N(t_0) - \mu N(t_0)$

$N(t_1) = N(t_0) \cdot (1 - \mu + \lambda) + \lambda N(t_0)$

$$N(t_1) = (1-\lambda+\mu)N(t_0)$$

$$N(t_2) = N(t_1)\,(1-\lambda+\mu)$$

$$= (1-\lambda+\mu)(1-\lambda+\mu)N(t_0)$$

$$= (1-\lambda+\mu)^2 N(t_0)$$

Induction (Proof)

$$\boxed{N_{n+1} = (1+\lambda+\mu)^n N_0}$$

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$$N_{n+1} = (1+\lambda-\mu)^n n_0$$

Cases:

(1) $1+\lambda-\mu = 1 \Longrightarrow \lambda-\mu=0$ or $\lambda=\mu$

$(1+\lambda-\mu)^n = 1$

$N_{n+1} = N_0$

(2) $0<(1+\lambda-\mu)<1$

$N_{n+1} \longrightarrow 0$

(3) $(1-\mu+\lambda) > 1$

$N_n \longrightarrow \infty$

$r \equiv \lambda-\mu$

net reproductive rate

HW: $\begin{cases} N_{n+1} = \underbrace{(1-\mu+\lambda)}_{b} N_n \\ N(0) = N_0 \end{cases}$

Ask what happens if we have a constant # of immigrant bugs coming in each year. Call that # c

$$\begin{cases} N_{n+1} = bN_n + c \\ N(0) = N_0 \end{cases}$$

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Hint:

$$N(0) = N_0$$

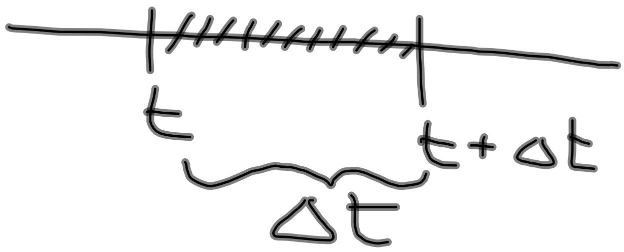
$$N(1) = bN_0 + c$$

$$\begin{aligned} N(2) &= bN(1) + c \\ &= b[bN_0 + c] + c \\ &= b^2 N_0 + bc + c \end{aligned}$$

$$\begin{aligned} N(3) &= bN(2) + c \\ &= b[b^2 N_0 + bc + c] + c \\ &= b^3 N_0 + b^2 c + bc + c \end{aligned}$$

. . . . .

$$N(t+\Delta t) = N(t) + \lambda N(t)\Delta t - \mu N(t)\Delta t$$



$$= N(t) + \lambda N(t)\Delta t - \mu N(t)\Delta t$$

$$N(t+\Delta t) - N(t) = \lambda \Delta t\, N(t) - \mu \Delta t\, N(t)$$

$$\frac{N(t+\Delta t) - N(t)}{\Delta t} = (\lambda - \mu) N(t)$$

$$\frac{dN}{dt} = (\lambda - \mu) N(t)$$

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$$\begin{cases} \dfrac{dN}{dt} = (\lambda - \mu)N, & \lambda - \mu \equiv r \\ N(0) = N_0 \end{cases}$$

$$\frac{dN}{dt} = rN$$

$$\int \frac{dN}{N} = \int r\,dt$$

$$\ln|N| = r\int dt = rt + C$$

$$\ln|N| = rt + C$$

$$e^{\ln|N|} = e^{rt+C}$$

$$N(t) = e^{rt+C}$$

$$= e^{rt}e^{C} = Ce^{rt}$$

$$N(t) = Ce^{rt}$$

$$N(0) = N_0 = C\underbrace{e^{r0}}_{1} = C$$

$$\boxed{N(t) = N_0 e^{rt}}$$

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Discrete

$$N(n) = (1+\underbrace{\lambda-\mu}_{r})^n N_0$$

$$= N_0 (1+r)^n$$

$$(1+r)^n = e^{\ln(1+r)^n}$$

$$= e^{n \ln(1+r)}$$

$$N(n) = N_0 \, e^{n \ln(1+r)}$$

Continuous

$$N(t) = N_0 e^{(\lambda-\mu)t}$$

$$N(t) = N_0 e^{rt}$$

$$N(t) = N_0 e^{tr}$$

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