Modeling

2 classes (general)

→ Simulation

→ Analytical (Equations)

→ Linguistic

→ Fuzzy set models

Analytical Models

		Space	
		yes	no
Time	discrete		
	continuous		

randomness vs no randomness

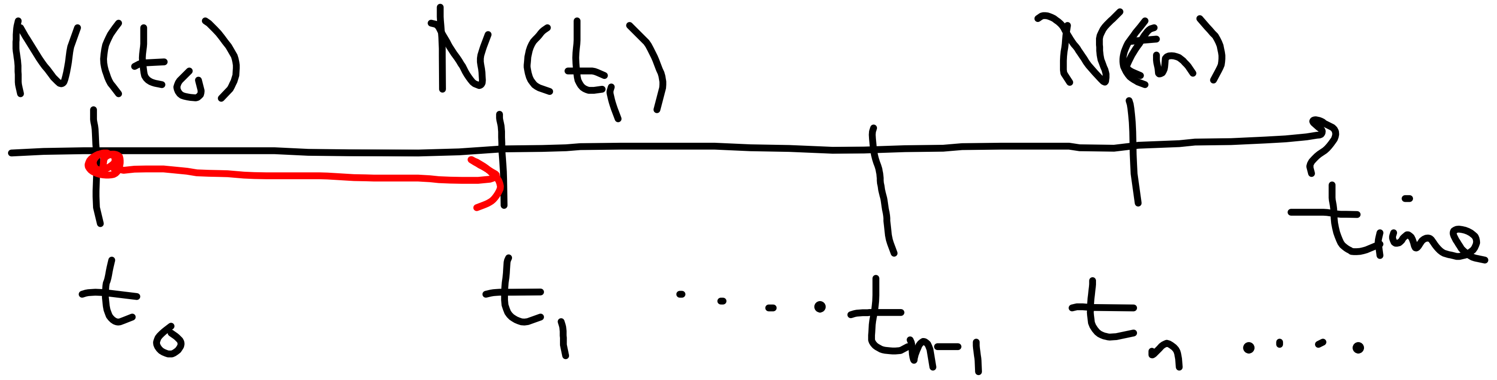
system of equations

Type

- Sir Robert May
- Hassell

Discrete time

Homogeneous population



$N(t_0) = N_0$ (given)

Δ time

$$N(t_1) = N(t_0) + \lambda N(t_0)\Delta t$$

birth / time

time

$$= N(t_0) + \lambda N(t_0) - \mu N(t_0)$$

$$= N(t_0) + (\underbrace{\lambda - \mu}_{= r})N(t_0)$$

$$N(t_1) = (1+r)N(t_0)$$

Parameter Estimation

$$N(t_2) = N(t_1) + rN(t_1)$$

$$= (1+r)N(t_1) = (1+r)(1+r)N(t_0)$$

$$\vdots$$

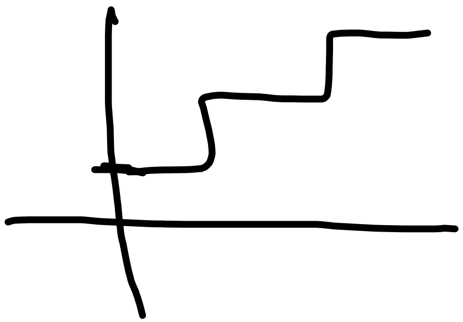
$$\boxed{N(t_n) = (1+r)^n N(t_0)}$$

continuous

$$N(t) = N_0 e^{rt}$$

$r > 0$ $\quad (1+r) > 1 \quad (1+r)^n \to \infty$

$n \to$ large



$$e^{\ln(1+r)^n} = e^{n\ln(1+r)}$$

	Discrete Time	Continuous Time
	$N(n) = (1+r)^n N_0$	$N(t) = N_0 e^{rt}$
$r=0$	$N(n) = N_0$	$N(t) = N_0$
$r \in [-1, 0)$	$N \to 0$	$N \to 0$
$r \in (-\infty, -1)$	$(-)^n \lvert 1+r \rvert^n N_0$	$N \to 0$

Bifurcation

biologically unfeasible for $n = 2m+1$

$m = 0, 1, \ldots$

Conjugacy Class

• Sensitivity to initial condition

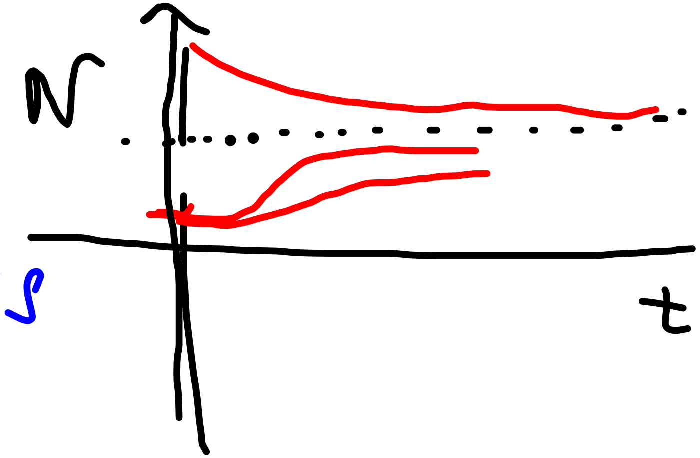
$\Longrightarrow$ chaotic behavior

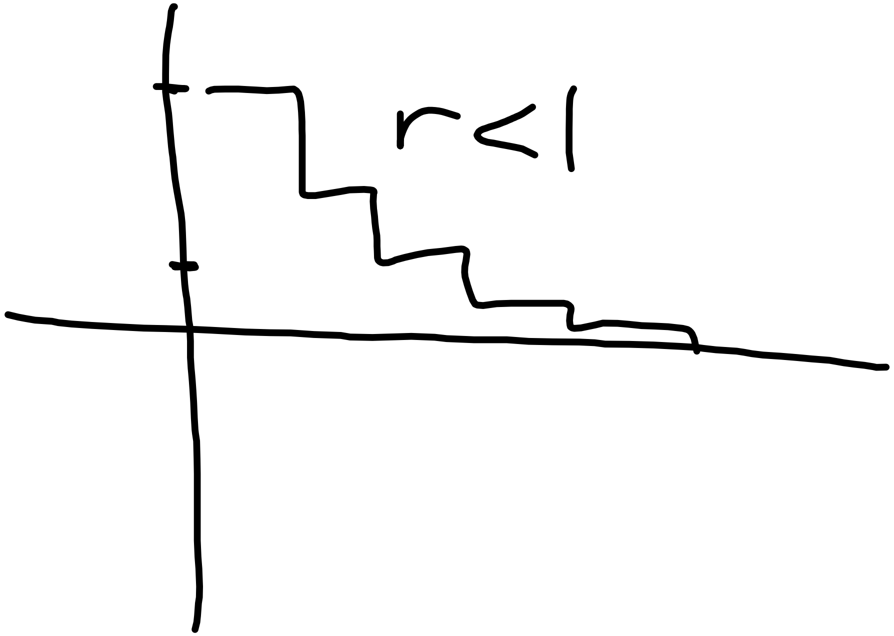
<u>Discrete</u>

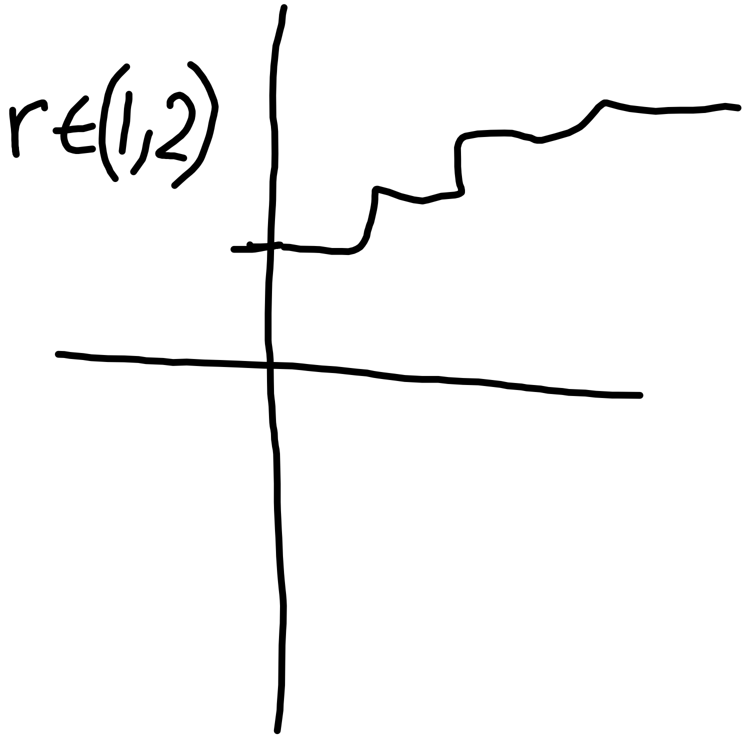
$$N_{n+1} = r N_n (1 - N_n) \qquad \frac{dN}{dt} = rN(1-N)$$

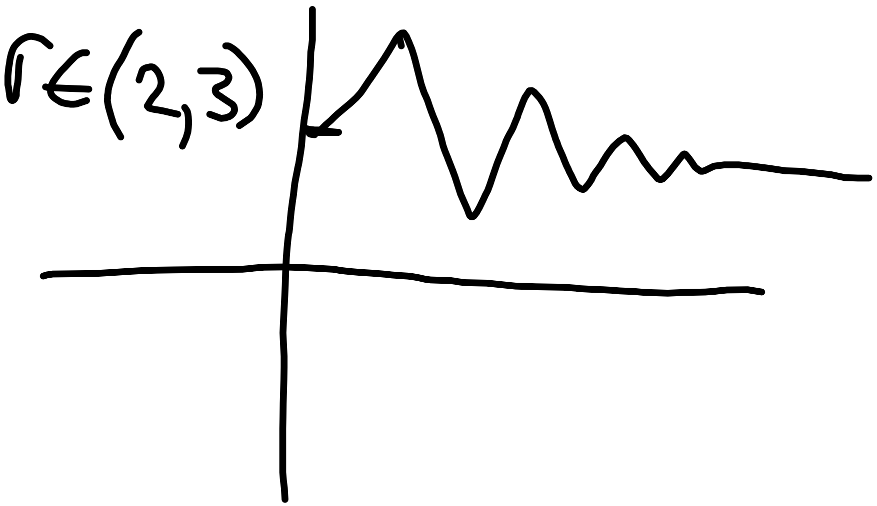
$\rightarrow$ cobwebbing analysis

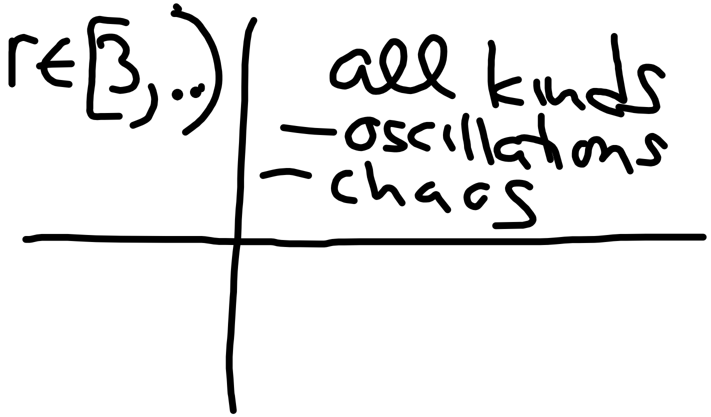
Sensitivity Analysis




r<1


r∈(1,2)


r∈(2,3)

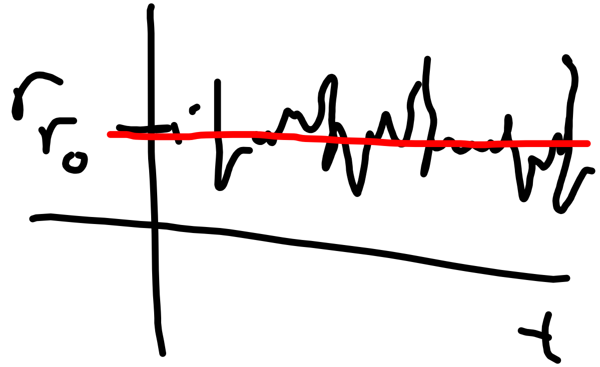

r∈[3,..)
all kinds
– oscillations
– chaos

Stochastic Models

- stochastic gene expression
- $1/f$ noise, white noise, colored noise

Cell Growth Model

$$\begin{cases} \dfrac{dN(t)}{dt} = rN(t) \\ N(t_0) = N_0 \end{cases}$$



Suppose my noise is $\gamma(t)$

But Suppose $r(t) = r_0 + \gamma(t)$

Want Average $\bar{r}(t) = r_0$ so

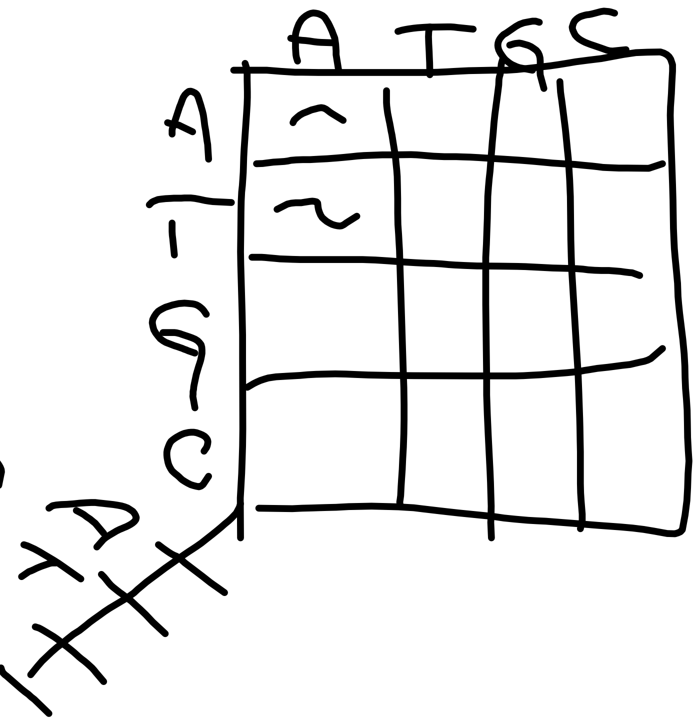
Probability Models

— Models that describe transitions between states of being.

Genes

Michael Waterman

Sam Karlin



Entropy

Information Measures

heat map

$$\boxed{S} \longrightarrow \boxed{I}$$

$$\boxed{\frac{dS}{dt} = -kSI}$$

contact rate

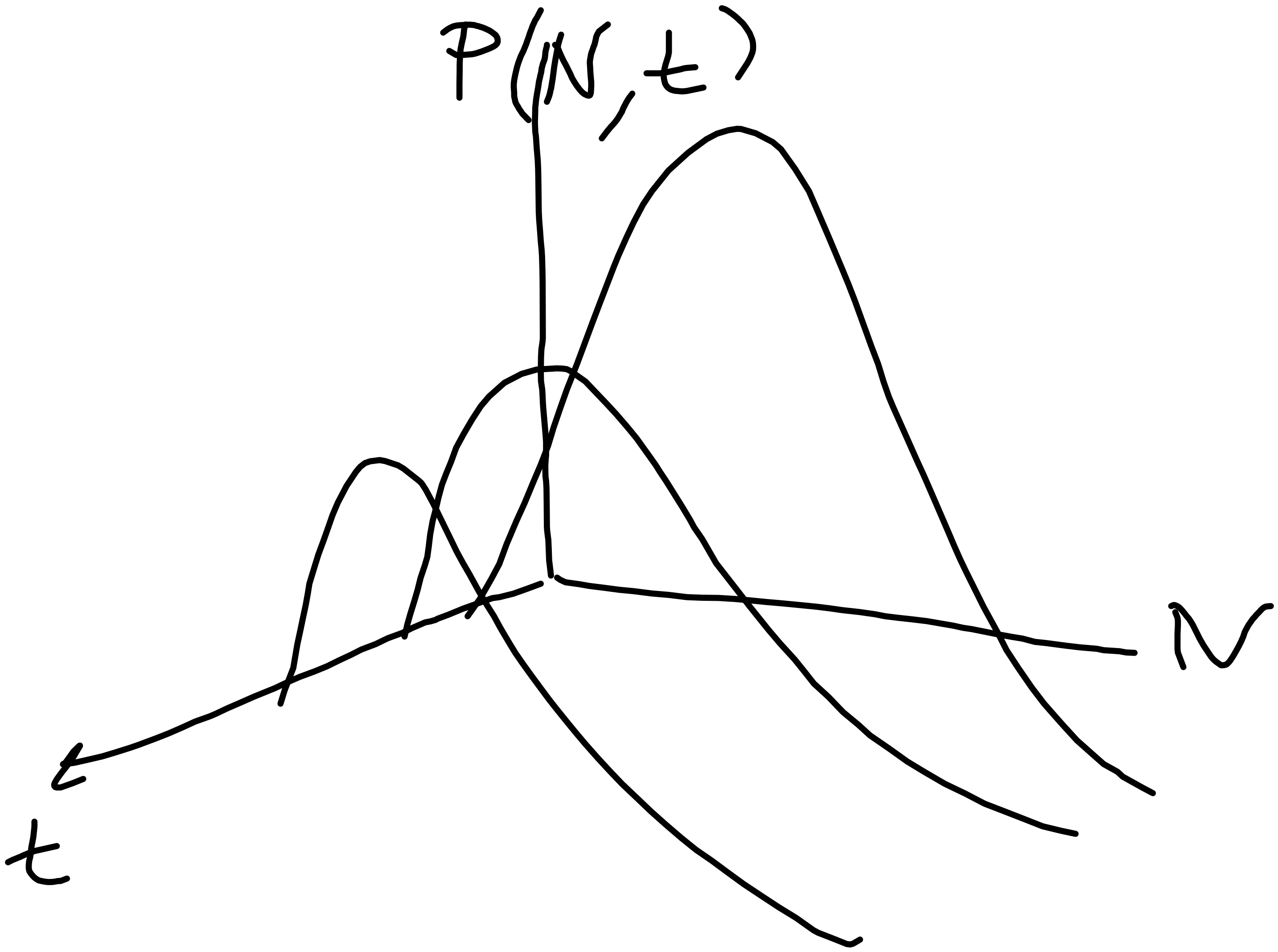
$$S(t+\Delta t) = S(t) - k\Delta t\, SI$$

$$P_I(t|S)$$

$$S + I = N_0$$

$$I = N_0 - S$$

$$\frac{dS}{dt} = -kS(N_0 - S)$$


P(N, t)
N
t

$$\text{Ave}(r(t)) = \text{Ave}(r_0 + \gamma(t))$$

$$= \text{Ave}(r_0) + \text{Ave}(\gamma(t))$$

$$r_0 = r_0 + \text{Avg}(\gamma(t))$$

Write $E[\gamma(t)] = 0$

$$\text{Var}[\gamma(t)] = \sigma^2$$